# MMSE Design of Redundant FIR Precoders for Arbitrary Channel Lengths

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***Abstract*—In this paper, the joint design of transmitter and receiver for multichannel data transmission over dispersive channels is considered. The design criterion is the minimization of the mean squared error (MSE) at the receiver output under the constraint of a fixed transmit power. The focus is on the practically important case where the transmitter employs finite impulse response (FIR) filters, and the channel impulse response has arbitrary length. The proposed algorithm allows a straightforward transmitter design and generally yields near-optimal solutions for the transmit filters. Under certain conditions, the exact solutions for optimum block transmission, as known from the literature, are obtained.**

***Index Terms*—Joint transmitter and receiver design, overlap block transmitter, redundant precoding.**

## I. Introduction

It is well known that redundancy introduced in the transmitter of a communication system may allow us to overcome serious intersymbol interference (ISI) problems due to highly dispersive channels. The process of shaping the transmit signal and/or introducing redundancy based on the knowledge of the channel is known as precoding. Various strategies have been followed in the design of precoders. Classical techniques such as Thomlinson–Harashima precoding use modulo arithmetic to manipulate the stream of transmit symbols [1], [2]. Recently studied linear techniques use a joint design of the transmit and receive filters. We are interested in the second category.

Prominent examples of redundant transmission techniques are discrete multitone modulation (DMT) and orthogonal frequency division multiplexing (OFDM), where a guard interval in form of a cyclic prefix is introduced [3]–[5]. With DMT and OFDM, ISI can be completely avoided if the channel is finite impulse response (FIR), and the length of the prefix is equal or larger than the channel order. Apart from (possibly applied) adaptive loading in the transmitter, the only adaptation of the transmitter to the channel is the choice of the length of the prefix. If the length of the channel impulse response exceeds the guard interval, however, the performance of DMT and OFDM degrades rapidly, and ISI and inter-channel interference (ICI, crosstalk) will occur. To cope with longer channel impulse responses, one can increase the length of the guard interval, but this will decrease the efficiency, as fewer data symbols can be transmitted. Increasing both the length of the guard interval and the number of subchannels allows one to maintain a desired bandwidth efficiency, but this strategy also has its limits. For example, the delay between transmitter and receiver may become unacceptably high. In addition, the hardware requirements increase with an increasing number of subchannels. Finally, channels that can be regarded as slowly time-varying when the number of subchannels is low may turn into fast time-varying ones if the number of subchannels and, thus, the lengths of the transmit and receive filters are significantly increased. Other approaches based on multirate filterbanks have been proposed in [6] and [7]. With these methods, symbol overlap is introduced even for ideal channels, and redundancy is introduced either in the frequency or in the time domain. The introduced redundancy then allows for better equalization on the receiver side.

The best performance can be expected when the transmitter and receiver impulse responses are entirely adapted to the channel. In recent years, this joint design problem has attracted numerous researchers, as it has the potential to yield very high throughput through dispersive channels without the need of costly algorithms such as maximum likelihood sequence estimation with the Viterbi algorithm. Salz [8] provided a first solution to the joint filter design problem, but it required the filters to have support within the first Nyquist zone $[-1/2T, 1/2T]$. Yang and Roy proposed an algorithm for the design of precoders that use excess bandwidth to introduce redundancy [9]. However, their method required an iteration to find the optimum solution. Xia studied the existence of redundant precoders that allow a perfect inversion of FIR channels with FIR receivers [10]. The effects of noise were not considered. In [11], Xia presented another suboptimal zero forcing (ZF) solution where a partially channel adapted, orthogonal block receiver is chosen first, and then, the best transmit filters for the given suboptimal receiver are designed. Scaglione *et al.* provided direct solutions to the joint design problem for the case of block transforms where the channel order does not exceed the length of an introduced guard interval of zeros [12], [13]. The optimality criteria considered are the the ZF and minimum mean squared error (MMSE) criteria [12] and the maximization of mutual information [13]. Mutual information has also been considered in [14] for a similar setting. Because the length of the guard interval in the block transforms of [12]–[14] is equal to the length of the cyclic prefix in DMT and OFDM, the same delay and bandwidth efficiency problems occur as with DMT or OFDM when the

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channel impulse response becomes long. Li and Ding provided a direct solution to the problem of minimizing the mean squared error (MSE) under the power constraint that allows arbitrary channel lengths [15]. However, the practical use of their exact solution is somewhat restricted because it turns out that both the ideal transmit and receive filters are generally IIR filters. Finally, transmitter design methods for the case where decision feedback receivers are employed have been proposed in [16]–[18].

In this paper, we are interested in the design of FIR precoders for the case where the channel impulse response may have arbitrary length. Note that this configuration is of significant interest for practical applications because real-world channel impulse responses may become extremely long, and the use of sufficiently long guard intervals, as required for DMT, OFDM, or the methods in [12]–[14], may be prohibitive due to delay constraints. The proposed design method considers the optimal receive filters for given transmit filters and channel, but during transmitter optimization, it uses an approximation for simplifying the objective function. For $L \leq N - M$, where $L$ is the channel order, $M$ is the number of subchannels, and $N$ is the upsampling factor in the transmitter, the algorithm yields the exact optimum solutions of [12], and for $L > N - M$, it leads to near-optimum solutions. The approach can be seen as an extension of the work in [12] from block transmission to overlapped block transmission.

The paper is organized as follows. Section II describes the input-output relationships of the considered transmit/receive system. Section III then addresses the design of optimal transmit and receive filters according to the MMSE criterion under the transmit power constraint. Section IV demonstrates the properties of the proposed algorithm in several examples, and finally, Section V gives some conclusions.

*Notation:* Vectors and matrices are printed in boldface. The superscript $\{\cdot\}^H$ means transposition and complex conjugation of a matrix or vector. $E\{\cdot\}$ denotes the expectation operation, and $\delta_{i,k}$ denotes the Kronecker symbol. $\mathrm{tr}\{\cdot\}$ is the trace and $\|\cdot\|_F$ is the Frobenius norm of a matrix. $\mathbb{C}$ denotes the set of complex numbers.

## II. Input–Output Relationships of the Transmit/Receive System

We consider the block diagram of a redundant precoder depicted in Fig. 1. The input stream $d(i)$ is split into $M$ parallel streams that are then upsampled by a factor of $N \geq M$ and fed into the $M$ transmit filters $g_k(n), k = 0, 1, \dots, M-1$. The sum of the filter output signals is the transmit signal $s(n)$. The amount of redundancy introduced by the transmitter (precoder) is determined by the ratio $N/M$. The transmit signal $s(n)$ is fed into a noisy channel with impulse response $c(n)$. The additive noise process $\eta(n)$ is assumed to be wide-sense stationary. On the receiver side, the signal $r(n)$ is filtered with the analysis filters $h_k(n), k = 0, 1, \dots, M-1$ and subsampled by $N$ to yield the parallel output data $\hat{d}_k(m)$. Finally, a parallel-to-serial conversion is used to obtain the output sequence $\hat{d}(i)$.

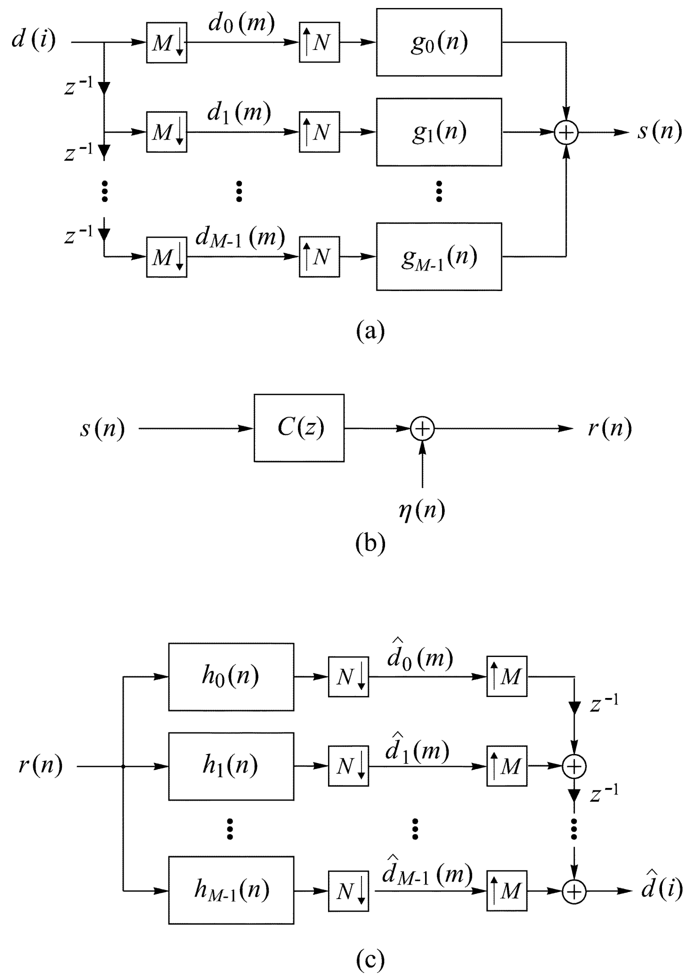



Fig. 1. Redundant precoder. (a) Transmitter. (b) Channel. (c) Receiver.

For the analysis of precoders, it is advantageous to decompose the filters into their polyphase components and to describe the system as a multiple-input–multiple-output (MIMO) system, as depicted in Fig. 2. The input vector to the MIMO system at time $m$ is given by $\boldsymbol{d}(m) = [d_0(m), d_1(m), \dots, d_{M-1}(m)]^T$ with $d_k(m) = d(mM - k)$. The output process, which is denoted as $\hat{\boldsymbol{d}}(m)$, has a similar definition. The transmit filterbank can be described via its $N \times M$ polyphase matrix [19]

$$\boldsymbol{G}(z) = \begin{bmatrix} G_{00}(z) & \cdots & G_{M-1,0}(z) \\ \vdots & & \vdots \\ G_{0,N-1}(z) & \cdots & G_{M-1,N-1}(z) \end{bmatrix} \quad (1)$$

where $G_{k,\ell}(z)$ is the $\ell$th type-1 polyphase component of the $k$th transmit filter, given by $G_{k,\ell}(z) = \sum_n g_k(nN + \ell)\, z^{-n}$. The polyphase matrix of the receiver filterbank is defined as

$$\boldsymbol{H}(z) = \begin{bmatrix} H'_{00}(z) & \cdots & H'_{0,N-1}(z) \\ \vdots & & \vdots \\ H'_{M-1,0}(z) & \cdots & H'_{M-1,N-1}(z) \end{bmatrix} \quad (2)$$

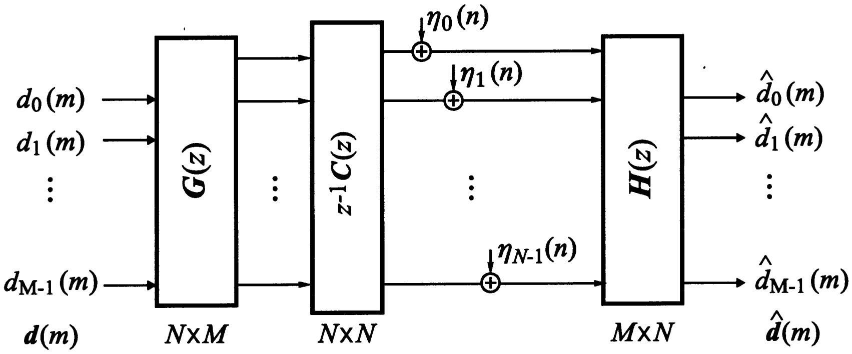



Fig. 2. Redundant precoder in polyphase (MIMO) representation.

with $H'_{k,\ell}(z) = \sum_n h_k(nN + N - 1 - \ell)z^{-n}$. Finally, the channel can be described via the pseudo-circulant $N \times N$ matrix

$$\boldsymbol{C}(z) = \begin{bmatrix} C_0(z) & z^{-1}C_{N-1}(z) & \cdots & z^{-1}C_1(z) \\ C_1(z) & C_0(z) & \cdots & z^{-1}C_2(z) \\ \vdots & & \ddots & \vdots \\ C_{N-1}(z) & C_{N-2}(z) & \cdots & C_0(z) \end{bmatrix} \quad (3)$$

where $C_\ell(z) = \sum_n c(nN + \ell)\, z^{-n}$. Alternatively, $\boldsymbol{C}(z)$ can be written as a polynomial of matrices as $\boldsymbol{C}(z) = \sum_n z^{-n}\boldsymbol{C}_n$. The often-desired property $\hat{\boldsymbol{d}}(n) = \boldsymbol{d}(n - n_0)$ is obtained in the noise-free case if $\boldsymbol{H}(z)$ and $\boldsymbol{G}(z)$ are chosen such that the perfect reconstruction (PR) condition

$$\boldsymbol{H}(z)\boldsymbol{C}(z)\boldsymbol{G}(z) = z^{-n_0+1}\boldsymbol{I}_{M\times M} \quad (4)$$

holds. Note that conditions on the channel $c(n)$ and the parameters $M$ and $N$ under which (4) can be satisfied have been studied in [10] and [12].

## III. Design Algorithm for MMSE Precoders

In the following, we first describe the assumptions made for the data and noise processes, and then, we explain the precoder design step by step.

### A. Assumptions on Data and Noise

The data process $d(n)$ is assumed to be white, zero-mean, wide-sense stationary and with variance $\sigma_d^2$. Colored data processes may be transferred into white ones via prefiltering so that the assumption of white data means no loss of generality. The noise process $\eta(n)$ is assumed to be zero mean, wide-sense stationary, and statistically independent of the data. It may be colored or white and can be described via its power spectral density matrix $\boldsymbol{S}_{\eta\eta}(e^{j\omega})$, which is given by

$$\boldsymbol{S}_{\eta\eta}(e^{j\omega}) = \sum_m \boldsymbol{R}_{\eta\eta}(m)e^{-j\omega m} \quad (5)$$

with

$$\boldsymbol{R}_{\eta\eta}(m) = E\{\boldsymbol{\eta}(n)\boldsymbol{\eta}^H(n+m)\}. \quad (6)$$

### B. Error Criterion

The aim in the design of MMSE precoders is to find the transmit and receive filters $\boldsymbol{G}(z)$ and $\boldsymbol{H}(z)$ such that the overall MSE

$$\text{MSE}_0 = E\{\|\boldsymbol{e}(n)^2\} = \text{tr}\{\boldsymbol{R}_{ee}(0)\}$$

with $\boldsymbol{e}(n) = \hat{\boldsymbol{d}}(n) - \boldsymbol{d}(n - n_0)$ and $\boldsymbol{R}_{ee}(m) = E\{\boldsymbol{e}(n)\boldsymbol{e}^H(n+m)\}$ is minimized under the condition of a fixed transmit power. Using Parseval's theorem, the MSE can alternatively be expressed via an integration over the trace of the power spectral density matrix

$$\boldsymbol{S}_{ee}(e^{j\omega}) = \sum_n \boldsymbol{R}_{ee}(n)e^{-j\omega n}.$$

The minimum MSE then becomes

$$\text{MSE}_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{tr}\{\boldsymbol{S}_{ee}(e^{j\omega})\}\, d\omega. \quad (7)$$

This is the expression for the MSE that will be used in the following derivations.

### C. Choice of Transmitter Structure

In this work, to allow for low latency time, the transmit polyphase matrix is chosen as a block of size $N \times M$

$$\boldsymbol{G}(z) = \boldsymbol{G}_0. \quad (8)$$

The only further restriction imposed on $\boldsymbol{G}_0$ is the power constraint

$$\sigma_d^2\, \text{tr}\{\boldsymbol{G}_0\boldsymbol{G}_0^H\} = P_0. \quad (9)$$

Thus, unlike in [12], the structure of $\boldsymbol{G}_0$ is not influenced by the length of the channel impulse response.

### D. MSE Under the Condition of Optimal Receive Filters

For any arbitrary matrix $\boldsymbol{G}_0$ of appropriate size and a given channel impulse response $c(n)$, the optimal MMSE receive filters can be found in a straightforward manner. In our case, the optimal polyphase matrix of the receive filters becomes[1]

$$\boldsymbol{H}(z) = z^{-n_0+1}\sigma_d^2 \times \left[\boldsymbol{I}_{M\times M} + \sigma_d^2\boldsymbol{G}_0^H\tilde{\boldsymbol{C}}(z)\boldsymbol{S}_{\eta\eta}^{-1}(z)\boldsymbol{C}(z)\boldsymbol{G}_0\right]^{-1} \times \boldsymbol{G}_0^H\tilde{\boldsymbol{C}}(z)\boldsymbol{S}_{\eta\eta}^{-1}(z) \quad (10)$$

where $\tilde{\boldsymbol{C}}(z)$ is the paraconjugate of $\boldsymbol{C}(z)$ given by $\tilde{\boldsymbol{C}}(z) = [\boldsymbol{C}(z)]^H$ for $|z| = 1$. In (10) and in the following derivations, it is assumed that $\boldsymbol{S}_{\eta\eta}^{-1}(z)$ exists. It is known from estimation theory that the role of the matrix $\boldsymbol{S}_{\eta\eta}^{-1}(z)$ in (10) is to implicitly

[1]Note that (10) is a straightforward frequency domain extension of the classical result $\boldsymbol{H} = [\boldsymbol{R}_{aa}^{-1} + \boldsymbol{S}^H\boldsymbol{R}_{nn}^{-1}\boldsymbol{S}]^{-1}\boldsymbol{S}^H\boldsymbol{R}_{nn}^{-1}$ for MMSE estimators based on the linear model $\boldsymbol{r} = \boldsymbol{S}\boldsymbol{a} + \boldsymbol{n}$, where $\boldsymbol{r}$ is the observation, $\boldsymbol{n}$ is noise, and $\boldsymbol{a}$ is the parameter vector to be estimated [20], [21].

whiten the additive noise. Thus, the assumption of the existence of $\boldsymbol{S}_{\eta\eta}^{-1}(z)$ is equivalent to the assumption of the existence of a noise whitening filter.

When employing the optimal receive filters according to (10), the power spectral density matrix of the estimation error becomes[2]

$$\boldsymbol{S}_{ee}(e^{j\omega}) = \sigma_d^2 \Big[\boldsymbol{I}_{M\times M} + \sigma_d^2 \boldsymbol{G}_0^H \tilde{\boldsymbol{C}}(e^{j\omega}) \times \boldsymbol{S}_{\eta\eta}^{-1}(e^{j\omega}) \boldsymbol{C}(e^{j\omega}) \boldsymbol{G}_0\Big]^{-1}. \quad (11)$$

*E. Transmitter Optimization*

The aim is now to find the matrix $\boldsymbol{G}_0$ that minimizes (7) with $\boldsymbol{S}_{ee}(e^{j\omega})$ according to (11) under the power constraint (9). Because the problem cannot (in general) be solved directly, we will provide an approximate solution. For this, we first describe the term $\tilde{\boldsymbol{C}}(e^{j\omega})\boldsymbol{S}_{\eta\eta}^{-1}(e^{j\omega})\boldsymbol{C}(e^{j\omega})$ in (11) as

$$\tilde{\boldsymbol{C}}(e^{j\omega})\boldsymbol{S}_{\eta\eta}^{-1}(e^{j\omega})\boldsymbol{C}(e^{j\omega}) = \sum_k \boldsymbol{\mathcal{R}}(k) e^{-j\omega k} \quad (12)$$

with

$$\boldsymbol{\mathcal{R}}(k) = \sum_\ell \sum_m \boldsymbol{C}_\ell^H \boldsymbol{K}_{\eta\eta}(m) \boldsymbol{C}_{k+\ell-m} \quad (13)$$

where $\boldsymbol{K}_{\eta\eta}(m)$ is the sequence of correlation matrices that corresponds to $\boldsymbol{S}_{\eta\eta}^{-1}(e^{j\omega})$

$$\boldsymbol{S}_{\eta\eta}^{-1}(e^{j\omega}) = \sum_m \boldsymbol{K}_{\eta\eta}(m) e^{-j\omega m}. \quad (14)$$

If the noise $\eta(n)$ is white with variance $\sigma_\eta^2$, then $\boldsymbol{S}_{\eta\eta}(e^{j\omega}) = \sigma_\eta^2 \boldsymbol{I}_{N\times N}$ and $\boldsymbol{K}_{\eta\eta}(m) = \delta_{m,0}\boldsymbol{I}_{N\times N}/\sigma_\eta^2$, and the above expressions simplify accordingly.

Using (12), the matrix $\boldsymbol{S}_{ee}(e^{j\omega})$ can be rewritten as

$$\boldsymbol{S}_{ee}(e^{j\omega}) = \sigma_d^2 \left[\boldsymbol{I}_{M\times M} + \sigma_d^2 \boldsymbol{G}_0^H \left[\sum_k \boldsymbol{\mathcal{R}}(k) e^{-j\omega k}\right] \boldsymbol{G}_0\right]^{-1}. \quad (15)$$

The idea for the approximation is to choose $\boldsymbol{G}_0$ from a subspace of $\mathbb{C}^{N,M}$ such that the terms

$$\boldsymbol{G}_0^H \boldsymbol{\mathcal{R}}(k) \boldsymbol{G}_0, \quad k \neq 0$$

become so small that they can be neglected in (15). Note that $\boldsymbol{G}_0^H \boldsymbol{\mathcal{R}}(k) \boldsymbol{G}_0$ for $k \neq 0$ represents the amount of interblock interference (IBI) between data stemming from blocks $\boldsymbol{d}(n)$ and $\boldsymbol{d}(n+k)$. To determine a suitable subspace for the choice of $\boldsymbol{G}_0$, we employ an iterative procedure. We do not explicitly formulate a basis for the required subspace and rather consider a projection $\boldsymbol{P}$ that projects onto the required subspace. The idea behind the method is to somewhat minimize the Frobenius norms $\|\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(k)\boldsymbol{P}\|_F$ for $k \neq 0$ while keeping $\|\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(0)\boldsymbol{P}\|_F$ mainly unchanged. The algorithm is as follows:.

Step 1) Initialize $\boldsymbol{P}$ as $\boldsymbol{P} = \boldsymbol{I}_{N\times N}$.

Step 2) Compute the eigenvectors $\boldsymbol{v}_N^{(k)}$ that correspond to the largest eigenvalues $\rho_N^{(k)}$ of the eigenvalue problems $[\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(k)\boldsymbol{P}[\boldsymbol{\mathcal{R}}(0)]^{-1}\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(k)\boldsymbol{P}]\boldsymbol{v}_i^{(k)} = \rho_i^{(k)}\boldsymbol{v}_i^{(k)}$ for all $k \neq 0$ for which $\boldsymbol{\mathcal{R}}(k) \neq \boldsymbol{0}$. Note that this can be efficiently done with the power method.

Step 3) Let $\boldsymbol{v}_{\max}$ be the eigenvector that belongs to the largest eigenvalue $\rho_{\max} := \max_k \rho_N^{(k)}$. If $\text{rank}(\boldsymbol{P}) > M$ and $\rho_{\max} > 0$, set

$$\boldsymbol{P} := \left[\boldsymbol{I}_{N\times N} - \frac{\boldsymbol{v}_{\max}\boldsymbol{v}_{\max}^H}{\boldsymbol{v}_{\max}^H\boldsymbol{v}_{\max}}\right]\boldsymbol{P}$$

and go back to Step 2. Otherwise, end the algorithm.

Given the projection matrix $\boldsymbol{P}$, the MSE (7) can be approximated as

$$\text{MSE}_1 = \sigma_d^2 \,\text{tr}\Big\{\big[\boldsymbol{I}_{M\times M} + \sigma_d^2 \boldsymbol{G}_0^H \boldsymbol{P}^H \boldsymbol{\mathcal{R}}(0) \boldsymbol{P} \boldsymbol{G}_0\big]^{-1}\Big\}. \quad (16)$$

Due to the inclusion of the projection matrix $\boldsymbol{P}$ in (16), we do not need to impose restrictions on $\boldsymbol{G}_0$ other than the power constraint (9). Minimizing $\text{MSE}_1$ will automatically lead to a matrix $\boldsymbol{G}_0$ that lies in the subspace onto which $\boldsymbol{P}$ projects.

Using the relationship

$$\text{tr}\{[\boldsymbol{I}_{M\times M} + \boldsymbol{A}\boldsymbol{B}^T]^{-1}\} = \text{tr}\{[\boldsymbol{I}_{N\times N} + \boldsymbol{B}^T\boldsymbol{A}]^{-1}\} - (N-M)$$

for matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of size $M \times N$, the $\text{svd}[\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(0)\boldsymbol{P}] = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$, and the fact that $\text{tr}\{\boldsymbol{A}\boldsymbol{B}\} = \text{tr}\{\boldsymbol{B}\boldsymbol{A}\}$, the MSE can be rewritten as

$$\text{MSE}_1 = \sigma_d^2 \,\text{tr}\Big\{\big[\boldsymbol{I}_{N\times N} + \sigma_d^2 \boldsymbol{\Lambda}\boldsymbol{Q}\big]^{-1}\Big\} - (N-M) \quad (17)$$

where $\boldsymbol{Q} = \boldsymbol{U}^H\boldsymbol{G}_0\boldsymbol{G}_0^H\boldsymbol{U}$. The power constraint (9) can be reformulated as $\sigma_d^2\text{tr}\{\boldsymbol{Q}\} = P_0$. As in [15] and according to Witsenhausen's result [22], the optimal matrix $\boldsymbol{Q}$ can be diagonal (i.e., $\boldsymbol{Q} = \text{diag}[q_1, q_2, \ldots, q_M]$), which simplifies the expression for the MSE to

$$\text{MSE}_1 = \sum_{i=1}^N \frac{1}{1+\sigma_d^2\lambda_i q_i} - (N-M). \quad (18)$$

The power constraint becomes $\sigma_d^2 \sum_{i=1}^N q_i = P_0$. The problem has now taken on the same form as in [12] and [15], and we can adapt the solution from there. Using the Lagrange multiplier technique and taking care of the fact that $q_i \geq 0$, we get

$$q_i = \max\left\{0, \frac{1}{\sigma_d^2\lambda_i}\left[\sqrt{\frac{\sigma_d^2\lambda_i}{\lambda}} - 1\right]\right\}, \quad i = 1, \ldots, N. \quad (19)$$

[2]Again, this is the frequency domain extension of the classical result $\boldsymbol{R}_{ee} = [\boldsymbol{R}_{aa}^{-1} + \boldsymbol{S}^H\boldsymbol{R}_{nn}^{-1}\boldsymbol{S}]^{-1}$ [20], [21].

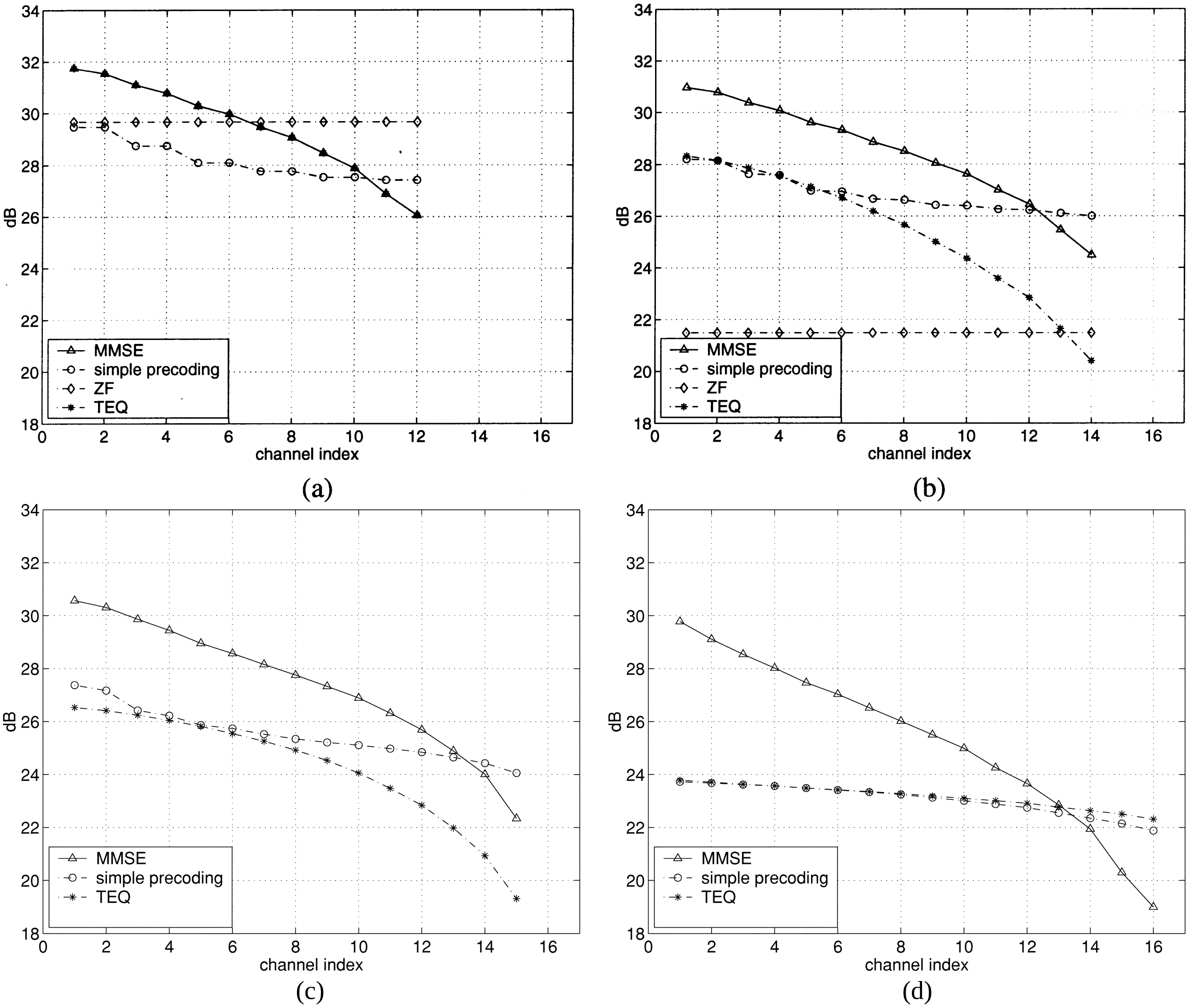



Fig. 3. Signal-to-noise ratios at receiver outputs. Parameters: $N = 16, L = 4, E_b/N_0 = 30$ dB. (a) $M = 12$. (b) $M = 14$. (c) $M = 15$. (d) $M = 16$.

Assuming that $\lambda_1, \ldots, \lambda_M > 0$ and that $q_1, \ldots, q_M$ belong to the $M$ channels with the highest SNRs, the Lagrange multiplier $\lambda$ can be determined from (19) and the power constraint

$$\lambda = \left[ \frac{\sum_{i=1}^{M} \left(\lambda_i \sigma_d^2\right)^{-1/2}}{P_0 + \sum_{i=1}^{M} \left(\lambda_i \sigma_d^2\right)^{-1}} \right]^2 . \tag{20}$$

Given the values $q_1, \ldots, q_M$, the required matrix $\boldsymbol{G}_0$ can be computed as

$$\boldsymbol{G}_0 = \boldsymbol{U}\boldsymbol{Q}^{1/2}. \tag{21}$$

It turns out that the transmit filters in $\boldsymbol{G}_0$ are the eigenvectors of $[\boldsymbol{P}^H \boldsymbol{\mathcal{R}}(0)\boldsymbol{P}]$ multiplied with the square roots of the transmit power factors $q_i$ for the individual subchannels. However, the solution (21) is not unique. Equivalent solutions with the same MSE can be easily derived by multiplying a given matrix $\boldsymbol{G}_0$ with arbitrary $M \times M$ unitary matrices from the right.

*1) Special Case $L \leq N - M$:* If the channel order $L$ is smaller or equal to $N - M$ and the noise is white, the proposed algorithm yields the exact solution for block transforms, as derived in [12]. This can be seen from the properties of the correlation matrices $\boldsymbol{\mathcal{R}}(k)$. Because the channel matrix $\boldsymbol{C}(z)$ reduces to $\boldsymbol{C}(z) = \boldsymbol{C}_0 + z^{-1}\boldsymbol{C}_1$, the matrices $\boldsymbol{\mathcal{R}}(k)$ are nonzero only for $k = -1, 0, 1$. $\boldsymbol{\mathcal{R}}(0)$ has rank $N$, whereas $\boldsymbol{\mathcal{R}}(-1)$ and $\boldsymbol{\mathcal{R}}(1)$ only have rank $L$. Thus, after $L$ iterations, the algorithm will lead to $\boldsymbol{P}^H \boldsymbol{\mathcal{R}}(k)\boldsymbol{P} = \boldsymbol{0}$ for $k \neq 0$, which means that all IBI will be canceled. The structure of $\boldsymbol{P}$ can be seen from the fact that $\boldsymbol{C}_1$ is nonzero only in the first $L$ rows and the last $L$ columns. Therefore, the algorithm yields

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{I}_{(N-L)\times(N-L)} & \boldsymbol{0}_{(N-L)\times L} \\ \boldsymbol{0}_{L\times(N-L)} & \boldsymbol{0}_{L\times L} \end{bmatrix}.$$

Because $\boldsymbol{G}_0$ has to lie in the subspace onto which $\boldsymbol{P}$ projects, we have

$$\boldsymbol{P}\boldsymbol{G}_0 = \boldsymbol{G}_0 = \begin{bmatrix} \boldsymbol{G}_{(N-L)\times M} \\ \boldsymbol{0}_{L\times M} \end{bmatrix}$$

with some matrix $\boldsymbol{G}_{(N-L)\times M}$. This structure of $\boldsymbol{G}_0$ corresponds to the trailing zero method of [12].

## IV. Examples

In this section, we will demonstrate the properties of the proposed precoder design method for two different scenarios. In the first one, we consider a configuration where the channel order $L$ is considerably smaller than the number of subchannels. In the second scenario, we look at a situation where the channel order is considerably higher than $N$, thus prohibiting the use of techniques that rely on a sufficiently long guard interval. Comparisons of the proposed method will be made with the following approaches.

1) *The channel-independent precoding technique of [10] that guarantees perfect inversion of the channel with FIR receive filters:* The transmit filters are simply $\boldsymbol{G}_0 = (P_0/M)^{-1/2}[\boldsymbol{I}_{M\times M}, \boldsymbol{0}_{(N-M)\times M}]^T$. We will refer to this method as "simple precoding."
2) The suboptimal ZF design of [11, Ch. 4.3.];
3) *A combination of time domain equalization (TEQ) and the MMSE block designs of [12]:* When the channel order exceeds the amount of introduced redundancy (i.e., $L > N - M$), the channel impulse response is first shortened to have effective length $N - M + 1$, thus resulting in virtually no IBI. Then, the MMSE block design of [12] is applied for the shortened channel response. The equalizer filters are designed according to the MMSE method in [23] and [24] and are placed on the transmitter side. The TEQ filter is taken into account when carrying out the power loading in order to ensure that the power constraint is exactly satisfied. The chosen TEQ length is 30.
4) *OFDM in cases where the introduced redundancy is sufficient to avoid IBI.*

The chosen parameters for the first setting are $L = 4$ and $N = 16$. The additive noise is assumed to be white, and the $E_b/N_0$ ratio is set to 30 dB. The channel impulse responses were chosen randomly, made up of $L + 1$ independent, zero-mean Gaussian random variables. Two hundred random channel realizations were considered in the experiments. All individual channel impulse responses have been normalized to have unit energy. Joint transmitter/receiver designs according the different methods have been carried out for $M = 12$, 14, 15, and 16. The SNRs were obtained for causal FIR receive filters that allow to detect the data with a total system delay of two blocks (i.e., $2N$ samples). The receive filter lengths were set to be $3N$. Similarly, the transmit filter lengths for the ZF method were also set to $3N$.

Fig. 3 shows the average SNRs at the receiver output for the various choices of $M$. As expected, the SNRs become higher with decreasing $M$. The comparison of Fig. 3(c) and (d) shows that already the introduction of a minimum amount of redundancy may yield a significant performance enhancement. Overall, the best performance is obtained with the proposed design algorithm. The TEQ method did not perform well in these experiments, except for $M = 12$, where the guard interval is sufficiently long, and no channel shortening takes place. Similarly, the simple, channel-independent precoder was several decibels below the optimized one. With the chosen filter length of $3N$, the ZF method had solutions only for $M = 12$ and $M = 14$, and the performance was good only for $M = 12$.

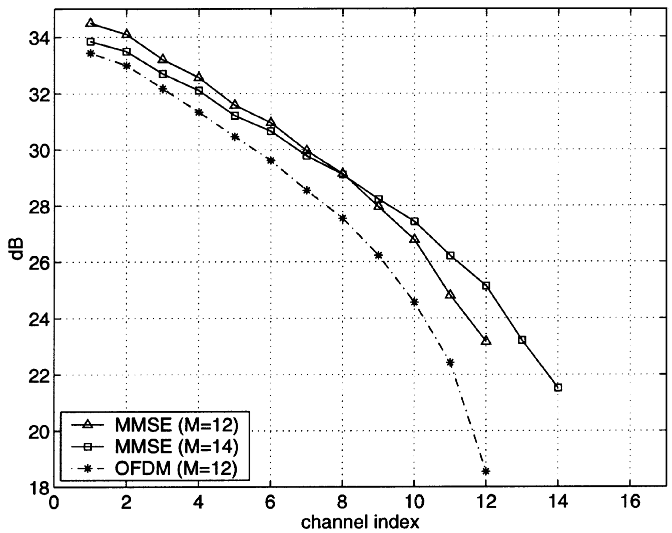
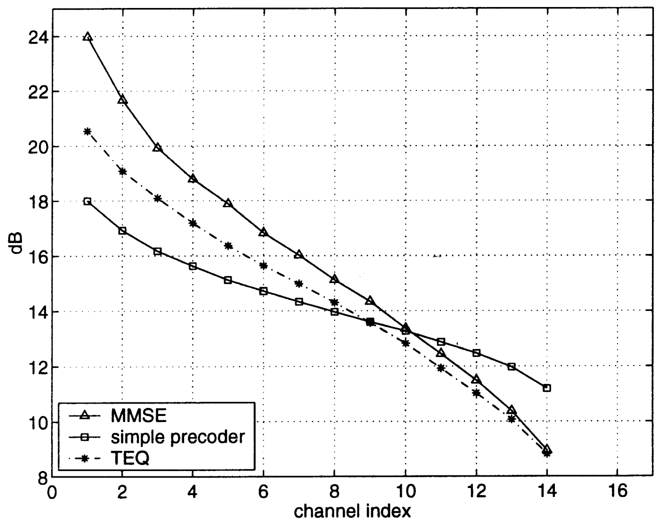



Fig. 4. Average SNRs at receiver outputs in descending order for precoders with $M = 12$ and $M = 14$ and for OFDM with $M = 12$. The OFDM channels have been sorted according to their SNRs before taking the average. Parameters: equal loading for all subchannels; equal total transmit power for all three cases; $E_b/N_0 = 30$ dB.

However, when increasing the transmit filter length from $3N$ to $5N$, the average SNR for the ZF method increased to 26.3 dB for $M = 14$, and solutions became available for $M = 15$ at an average SNR of 10.4 dB. When increasing the transmit filter length further up to $15N$, only marginal improvements could be recognized.

To compare the results with OFDM, the loading had been changed to equal power for all subchannels by choosing $\boldsymbol{Q}$ as a scaled identity matrix. Fig. 4 shows the SNRs for the proposed design with $M = 12$ and $M = 14$ and for OFDM with $M = 12$. The total transmit power is the same in all three cases. As the results show, the proposed transmit filters for both $M = 12$ and $M = 14$ yield a significant improvement over OFDM. For $M = 14$, the SNRs are not only better than for OFDM but also make two additional channels available.

For the second setting, channels of order $L = 20$ and block lengths $N = 16$ and $N = 32$ were chosen. With a block length of 16, this leads to IBI between three adjacent blocks, and even with $N = 32$, we have massive IBI between adjacent blocks. Note that both configurations cannot be treated with the algorithm of [12], and the algorithm of [15] would lead to IIR transmit filters. Further note that to allow for block transmission without IBI, one would have to increase $N$ substantially. However, this would introduce a large delay if an acceptable ratio $M/N$ was to be maintained. The proposed algorithm, on the other hand, is able to carry out the joint transmitter/receiver design with overlapping blocks. In the experiments, white channel noise with an $E_b/N_0$ of 30 dB was assumed, and 200 randomly generated channels have been tested. SNRs were collected for causal MMSE FIR receive filters of length $3N$ that recover the data with a delay of $2N$. Fig. 5 shows a comparison of the average SNRs obtained with the proposed algorithm, the simple precoder of [10], and the TEQ method. For $N = 16$ and $M = 14$, the optimized transmitter has several subchannels with SNRs that are much better than for the simple one, but



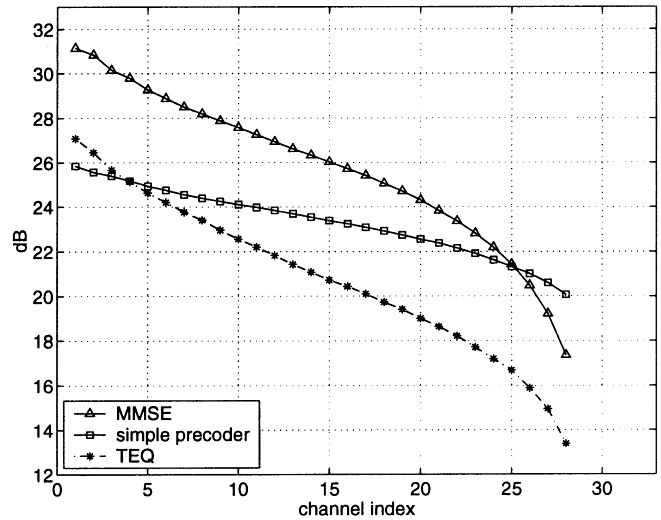




Fig. 5. Signal to noise ratios at receiver outputs. Parameters: $L = 20, E_b/N_0 = 30$ dB. (a) $N = 16, M = 14$. (b) $N = 32, M = 28$.

it also has some subchannels with worse performance. On average, both methods perform approximately equally well. The TEQ design, however, is somewhat behind the MMSE design for all subchannels. When increasing the block size to 32 and the number of subchannels to 28, the results significantly improve, as can be seen by comparing Fig. 5(a) and (b). The experiments show that the algorithm can handle massively dispersive channels and overlapped block transmission but also that it is advantageous to have IBI only between adjacent blocks.

## V. Conclusion

A method for the joint design of transmitter and receiver for data transmission over dispersive channels has been presented. The proposed method can treat the practically important case where the transmitter is FIR and the channel length is so high that it causes a significant amount of IBI. The approach allows for low latency transmission over dispersive channels and can be seen as an extension of the work in [12] from block transmission to overlapped block transmission. Design examples have confirmed the effectiveness of the design method.

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