Design of Redundant FIR Precoders for Arbitrary Channel Lengths based on an MMSE Criterion

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***Abstract*—In this paper, the joint design of transmitter and receiver for multichannel data transmission over dispersive channels is considered. In particular, the practically important case where the transmitter consists of FIR filters and the channel impulse response has arbitrary length is addressed. The design criterion is the minimization of the mean squared error at the receiver output under the constraint of a fixed transmit power. The proposed algorithm allows a straightforward transmitter design and yields (in general) a near-optimal solution for the transmit filters. Under certain conditions, the exact solution for the optimal transmitter is obtained.**

I. Introduction

It is well known that redundancy introduced in the transmitter of a communication system may allow to overcome serious intersymbol interference (ISI) problems due to highly dispersive channels. The process of shaping the transmit signal and/or introducing redundancy based on the knowledge of the channel is also known as precoding. Various strategies have been followed in the design of precoders. Classical techniques such as Thomlinson-Harashima precoding use modulo arithmetic to manipulate the stream of transmit symbols [1, 2]. More recently studied linear techniques use a joint design of the transmit and receive filters. We are interested in the second category. Prominent examples of redundant transmission techniques that are somewhat matched to the channel are DMT and OFDM where a guard interval in form of a cyclic prefix is introduced [3, 4]. With DMT and OFDM, ISI can be completely avoided if the channel is FIR and the length of the prefix is equal or larger than the channel order. Apart from (possibly applied) adaptive loading in the transmitter the only adaptation of the transmitter to the channel is the choice of the length of the prefix. Better performance than with DMT or OFDM can be expected when the transmitter and receiver impulse responses are entirely adapted to the channel. In recent years this joint design problem has attracted numerous researchers, as it has the potential to yield very high throughput through dispersive channels without the need of costly algorithms such as maximum likelihood sequence estimation with the Viterbi algorithm.

Salz [5] provided a first solution to the joint filter design problem, but it required the filters to have support within the first Nyquist zone $[-1/2T, 1/2T]$. Yang and Roy proposed an algorithm for the design of precoders that use excess bandwidth to introduce redundancy [6]. However, their method required an iteration to find the optimum solution. Xia studied the existence of redundant precoders that allow a perfect inversion of FIR channels with FIR receivers [7]. The effects of noise were not considered. Scaglione et al. provided direct solutions to the joint design problem for the case of block transforms where the channel order does not exceed the length of an introduced guard interval of zeros [8,9]. The optimality criteria considered are the the zero forcing (ZF) and MMSE criteria [8] and the maximization of mutual information [9]. Because the length of the guard interval in the block transforms of [8, 9] is equal to the length of the cyclic prefix in DMT and OFDM, the same delay and bandwidth efficiency problems occur as with DMT or OFDM when the channel impulse response becomes long. Li and Ding provided a direct solution to the problem of minimizing the MSE under the power constraint which allows arbitrary channel lengths [10]. However, the practical use of their exact solution is somewhat restricted, because it turns out that both the ideal transmit and receive filters are generally IIR filters.

In this paper, we are interested in the design of precoders where the transmit filters are FIR and the channel may have arbitrary length. Note that this configuration is of significant interest for practical applications, because real channel impulse responses may become extremely long and the use of sufficiently long guard intervals, as required for DMT, OFDM, or the method in [8], may be prohibitive due to delay constraints. The proposed design method considers the optimal receive filters for given transmit filters and channel, but during transmitter optimization it uses an approximation for simplifying the objective function. For $L \leq N - M$, where L is the channel order, M is the number of subchannels, and N is the upsampling factor in the transmitter, the algorithm yields the exact optimum solutions of [8], and for $L > N - M$ it leads to near optimum solutions.

II. MIMO Description of Precoders

A block diagram of a redundant precoder is given in Fig. 1. The input stream $d(m)$ is split into M parallel streams which are then upsampled by a factor of $N \geq M$ and fed into the M transmit filters $g_k(n),\ k = 0, 1, \ldots, M-1$. The channel is described by its impulse response $c(n)$ and an additive, stationary noise process $\eta(n)$. The receive signal is filtered with the analysis filters $h_k(n)$, $k = 0, 1, \ldots, M-1$ and subsampled by N to

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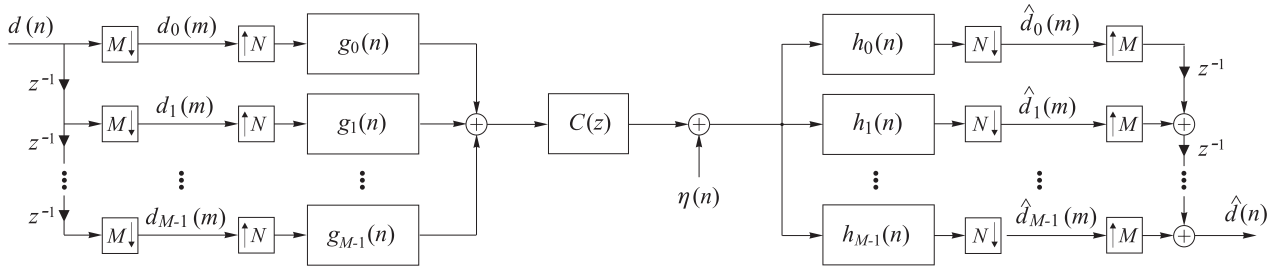

Fig. 1. Redundant precoder.

yield the parallel output data $\hat{d}_k(m)$. Finally, a parallel/serial conversion yields the output sequence $\hat{d}(n)$.

For the analysis of precoders it is advantageous to decompose the filters into their polyphase components and to describe the system as a multiple-input multiple-output (MIMO) system as depicted in Fig. 2. The input vector to the MIMO system at time m is given by $\boldsymbol{d}(m) = [d_0(m), d_1(m), \ldots, d_{M-1}(m)]^T$ with $d_k(m) = d(mM - k)$. The output process, denoted as $\hat{\boldsymbol{d}}(m)$, has a similar definition. The transmit filter bank can be described via its $N \times M$ polyphase matrix [11]

$$\boldsymbol{G}(z) = \begin{bmatrix} G_{00}(z) & \ldots & G_{M-1,0}(z) \\ \vdots & & \vdots \\ G_{0,N-1}(z) & \ldots & G_{M-1,N-1}(z) \end{bmatrix} \quad (1)$$

where $G_{k,\ell}(z)$ is the ℓth polyphase component of the kth transmit filter, given by

$$G_{k,\ell}(z) = \sum_n g_k(nN + \ell)\, z^{-n}. \quad (2)$$

Alternatively, $\boldsymbol{G}(z)$ may be expressed as $\boldsymbol{G}(z) = \sum_n \boldsymbol{G}_n z^{-n}$ with $[\boldsymbol{G}_n]_{\ell,k} = g_k(nN + \ell)$.

The polyphase matrix of the receiver filter bank is given by

$$\begin{aligned} \boldsymbol{H}(z) &= \begin{bmatrix} H'_{00}(z) & \ldots & H'_{0,N-1}(z) \\ \vdots & & \vdots \\ H'_{M-1,0}(z) & \ldots & H'_{M-1,N-1}(z) \end{bmatrix} \\ &= \sum_n \boldsymbol{H}_n z^{-n} \end{aligned} \quad (3)$$

with

$$\begin{aligned} H'_{k,\ell}(z) &= \textstyle\sum_n h_k(nN + N - 1 - \ell)\, z^{-n}, \\ [\boldsymbol{H}_n]_{k,\ell} &= h_k(nN + N - 1 - \ell). \end{aligned} \quad (4)$$

The channel can be described via the pseudo-circulant $N \times N$ matrix

$$\boldsymbol{C}(z) = \begin{bmatrix} C_0(z) & z^{-1}C_{N-1}(z) & \ldots & z^{-1}C_1(z) \\ C_1(z) & C_0(z) & \ldots & z^{-1}C_2(z) \\ \vdots & & \ddots & \vdots \\ C_{N-1}(z) & C_{N-2}(z) & \ldots & C_0(z) \end{bmatrix} \quad (5)$$

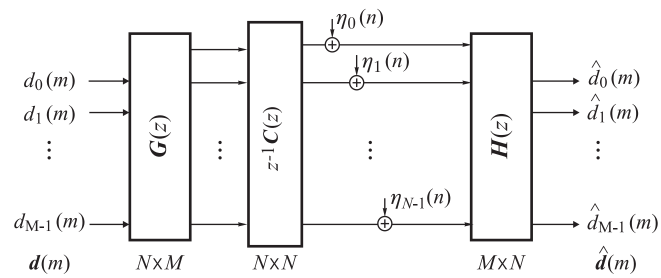

Fig. 2. Redundant precoder in polyphase (MIMO) representation.

with $C_\ell(z) = \sum_n c(nN + \ell)\, z^{-n}$.

The desired property

$$\hat{\boldsymbol{d}}(n) = \boldsymbol{d}(n - n_0) \quad (6)$$

is obtained in the noise free case if $\boldsymbol{H}(z)$ and $\boldsymbol{G}(z)$ are chosen such that the perfect reconstruction (PR) condition

$$\boldsymbol{H}(z)\, \boldsymbol{C}(z)\, \boldsymbol{G}(z) = z^{-n_0} \boldsymbol{I}_{M \times M} \quad (7)$$

holds. Conditions on the channel $c(n)$ and the parameters M and N under which (7) can be satisfied have been studied in [7, 8].

III. Design of MMSE Precoder

In the following we assume mutually independent, white, zero-mean data and noise processes with variances σ_d^2 and σ_η^2, respectively. The restriction to white processes is introduced to simplify the notation. The more general case with non-white data and noise processes can be derived from the presented algorithm through the introduction of whitening filters.

The aim in the design of MMSE precoders is to find the transmit and receive filters $\boldsymbol{G}(z)$ and $\boldsymbol{H}(z)$ such that the overall MSE

$$MSE_0 = E\left\{ \left\| \hat{\boldsymbol{d}}(n) - \boldsymbol{d}(n - n_0) \right\|^2 \right\}$$

is minimized under the condition of a fixed transmit power P_0. Using Parseval's theorem the MSE can alternatively be expressed via an integration over the trace of the power spectral density matrix $\boldsymbol{S}_{ee}(e^{j\omega})$ of the estimation error $\boldsymbol{e}(n) =$

$\hat{\boldsymbol{d}}(n) - \boldsymbol{d}(n-n_0)$:

$$MSE_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{tr}\left\{\boldsymbol{S}_{ee}(e^{j\omega})\right\} d\omega. \quad (8)$$

The definition of $\boldsymbol{S}_{ee}(e^{j\omega})$ will be given below.

In this work, to allow for minimal latency time the transmit polyphase matrix is chosen as a block of size $N \times M$:

$$\boldsymbol{G}(z) = \boldsymbol{G}_0. \quad (9)$$

The only further restriction imposed on $\boldsymbol{G}_0$ is the power constraint

$$\sigma_d^2 \, \text{tr}\left\{\boldsymbol{G}_0\boldsymbol{G}_0^H\right\} = P_0. \quad (10)$$

Thus, unlike in [8] the structure of $\boldsymbol{G}_0$ is not influenced by the length of the channel impulse response.

For any arbitrary matrix $\boldsymbol{G}_0$ of appropriate size and a given channel impulse response $c(n)$ the optimal MMSE receive filters can be found in a straightforward manner. In our case the optimal polyphase matrix of the receive filters becomes

$$\begin{aligned}\boldsymbol{H}(z) \;=\; & z^{-n_0}\,\sigma_d^2\left[\boldsymbol{I}_{M\times M} + \tfrac{\sigma_d^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\tilde{\boldsymbol{C}}(z)\boldsymbol{C}(z)\boldsymbol{G}_0\right]^{-1} \\ & \times \boldsymbol{G}_0^H\tilde{\boldsymbol{C}}(z)\end{aligned} \quad (11)$$

where $\tilde{\boldsymbol{C}}(z)$ is the paraconjugate of $\boldsymbol{C}(z)$ given by

$$\tilde{\boldsymbol{C}}(z) = [\boldsymbol{C}(z)]^H, \quad |z| = 1. \quad (12)$$

In the following we assume that the optimal receive filters according to (11) are employed. The power spectral density matrix of the estimation error then becomes[1]

$$\boldsymbol{S}_{ee}(e^{j\omega}) = \sigma_d^2\left[\boldsymbol{I}_{M\times M} + \frac{\sigma_d^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\tilde{\boldsymbol{C}}(e^{j\omega})\boldsymbol{C}(e^{j\omega})\boldsymbol{G}_0\right]^{-1}. \quad (13)$$

A similar expression was derived in [10].

The aim is now to find the matrix $\boldsymbol{G}_0$ that minimizes (8) with $\boldsymbol{S}_{ee}(e^{j\omega})$ according to (13) under the power constraint (10). Because the problem cannot (in general) be solved directly, we will provide an approximate solution. To point out the approximations made, we describe the term $\tilde{\boldsymbol{C}}(e^{j\omega})\boldsymbol{C}(e^{j\omega})$, which has the form of an energy density matrix, as the Fourier transform of its associated autocorrelation sequence:

$$\tilde{\boldsymbol{C}}(e^{j\omega})\boldsymbol{C}(e^{j\omega}) = \sum\nolimits_k \boldsymbol{\mathcal{R}}_{cc}(k)e^{-j\omega k} \quad (14)$$

with

$$\boldsymbol{\mathcal{R}}_{cc}(k) = \sum\nolimits_\ell \boldsymbol{C}_\ell^H\boldsymbol{C}_{\ell+k} \quad (15)$$

where

$$\boldsymbol{C}(z) = \sum\nolimits_\ell z^{-\ell}\,\boldsymbol{C}_\ell. \quad (16)$$

[1] Note that this is a straightforward frequency domain extension of the classical result $\boldsymbol{R}_{ee} = [\boldsymbol{R}_{aa}^{-1} + \boldsymbol{S}^H\boldsymbol{R}_{nn}^{-1}\boldsymbol{S}]^{-1}$ for MMSE estimators based on the linear model $\boldsymbol{r} = \boldsymbol{S}\boldsymbol{a} + \boldsymbol{n}$ where $\boldsymbol{r}$ is the observation, $\boldsymbol{n}$ is noise, and $\boldsymbol{a}$ is the parameter vector to be estimated.

Thus, $\boldsymbol{S}_{ee}(e^{j\omega})$ can be rewritten as

$$\begin{aligned}\boldsymbol{S}_{ee}(e^{j\omega}) \;=\; & \sigma_d^2\Bigg[\boldsymbol{I}_{M\times M} \\ & + \frac{\sigma_d^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\left[\sum_k \boldsymbol{\mathcal{R}}_{cc}(k)e^{-j\omega k}\right]\boldsymbol{G}_0\Bigg]^{-1}.\end{aligned} \quad (17)$$

The idea for the approximation is to choose $\boldsymbol{G}_0$ from a subspace of $\mathbb{C}^{N,M}$ such that the terms

$$\boldsymbol{G}_0^H\boldsymbol{\mathcal{R}}_{cc}(k)\boldsymbol{G}_0, \quad k \neq 0$$

become so small that they can be neglected in (17). Note that $\boldsymbol{G}_0^H\boldsymbol{\mathcal{R}}_{cc}(k)\boldsymbol{G}_0$ for $k \neq 0$ represents the amount of interblock interference (IBI) between data stemming from blocks $\boldsymbol{d}(n)$ and $\boldsymbol{d}(n+k)$ while $\boldsymbol{G}_0^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{G}_0$ represents the actual transmission through the channel. To determine a suitable subspace for the choice of $\boldsymbol{G}_0$ we employ an iterative procedure based on the singular value decomposition (svd). We do not explicitly formulate a basis for the required subspace, and rather consider a projection $\boldsymbol{P}$ that projects onto the required subspace. The algorithm is as follows:

Step 1: Let $\boldsymbol{P} = \boldsymbol{I}_{N\times N}$

Step 2: Compute the svd's

$$\boldsymbol{A}_k\boldsymbol{\Sigma}_k\boldsymbol{B}_k^H = \boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(k)\boldsymbol{P}$$

for all $k \neq 0$ for which $\boldsymbol{\mathcal{R}}_{cc}(k) \neq \mathbf{0}$.

Step 3: Determine the largest singular value for $k \neq 0$ and denote it as σ_{max}. Assuming that σ_{max} is contained in matrix $\boldsymbol{\Sigma}_K$ denote the corresponding column of $\boldsymbol{A}_K$ as $\boldsymbol{a}$.

Step 4: If rank($\boldsymbol{P}$) $> M$ and $\sigma_{max} > 0$ set

$$\boldsymbol{P} := \left[\boldsymbol{I}_{N\times N} - \boldsymbol{a}\boldsymbol{a}^H\right]\boldsymbol{P}$$

and go back to Step 2. Otherwise, end the algorithm.

The MSE (8) can now be approximated as

$$MSE_1 = \sigma_d^2\,\text{tr}\left\{\left[\boldsymbol{I}_{M\times M} + \frac{\sigma_d^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}\boldsymbol{G}_0\right]^{-1}\right\}. \quad (18)$$

The reason for including the projection matrix $\boldsymbol{P}$ in (18) instead of using a basis approach is that we do not need to impose restrictions on $\boldsymbol{G}_0$ other than the power constraint (10). Minimizing MSE_1 will automatically lead to a matrix $\boldsymbol{G}_0$ that lies in the subspace onto which $\boldsymbol{P}$ projects.

Using the relationship

$$\begin{aligned}\text{tr}\left\{\left[\boldsymbol{I}_{M\times M} + \boldsymbol{A}\boldsymbol{B}^T\right]^{-1}\right\} \;=\; & \text{tr}\left\{\left[\boldsymbol{I}_{N\times N} + \boldsymbol{B}^T\boldsymbol{A}\right]^{-1}\right\} \\ & -(N-M)\end{aligned}$$

for matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of size $M \times N$ the minimum MSE can be rewritten as

$$MSE_1 = \sigma_d^2 \operatorname{tr}\left\{\left[\boldsymbol{I}_{N\times N} + \frac{\sigma_d^2}{\sigma_\eta^2}\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}\boldsymbol{G}_0\boldsymbol{G}_0^H\right]^{-1}\right\} - (N-M). \tag{19}$$

Now we consider the svd $[\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}] = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$, insert it into (19), and rewrite the expression obtained using the fact that $\operatorname{tr}\{\boldsymbol{AB}\} = \operatorname{tr}\{\boldsymbol{BA}\}$. With the shorthand $\boldsymbol{Q} = \boldsymbol{U}^H\boldsymbol{G}_0\boldsymbol{G}_0^H\boldsymbol{U}$ this yields

$$MSE_1 = \sigma_d^2 \operatorname{tr}\left\{\left[\boldsymbol{I}_{N\times N} + \sigma_d^2\boldsymbol{\Lambda}\boldsymbol{Q}\right]^{-1}\right\} - (N-M). \tag{20}$$

The power constraint (10) can be reformulated as

$$\sigma_d^2 \operatorname{tr}\{\boldsymbol{Q}\} = P_0. \tag{21}$$

The aim is now to minimize (20) under the constraint (21). As in [10] and according to Wirtsenhausen's result [12] the optimal matrix $\boldsymbol{Q}$ can be diagonal, which simplifies our criterion to

$$MSE_1 = \sum_{i=1}^{N} \frac{1}{1+\sigma_d^2\lambda_i q_i} - (N-M), \tag{22}$$

and the power constraint becomes

$$\sigma_d^2 \sum_{i=1}^{N} q_i = P_0. \tag{23}$$

Using the Lagrange multiplier technique and taking care of the fact that $q_i \geq 0$ we get

$$q_i = \max\left\{0, \frac{1}{\sigma_d^2\lambda_i}\left[\sqrt{\frac{\sigma_d^2\lambda_i}{\lambda}} - 1\right]\right\}, \quad i = 1,\ldots,N. \tag{24}$$

We assume that $\lambda_1,\ldots,\lambda_M > 0$ and that $q_1,\ldots,q_M$ belong to the M channels with the highest SNR's. The Lagrange multiplier λ can then be computed from (24) and the power constraint $\sigma_d^2 \sum_{i=1}^{M} q_i = P_0$. It amounts to

$$\lambda = \left[\frac{\sum_{i=1}^{M}(\lambda_i\sigma_d^2)^{-1/2}}{P_0 + \sum_{i=1}^{M}(\lambda_i\sigma_d^2)^{-1}}\right]^2. \tag{25}$$

Given the values $q_1,\ldots,q_M$ the required matrix $\boldsymbol{G}_0$ can be computed as

$$\boldsymbol{G}_0 = \boldsymbol{U}\boldsymbol{Q}^{1/2}. \tag{26}$$

It turns out that the transmit filters in $\boldsymbol{G}_0$ are the eigenvectors of $[\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}]$ multiplied with the square roots of the transmit power factors q_i for the individual subchannels. However, the solution (26) is not unique. Equivalent solutions with the same MSE can be easily derived by multiplying a given matrix $\boldsymbol{G}_0$ with arbitrary $M \times M$ unitary matrices from the right.

The special case $L \leq N - M$:

If the channel order L is smaller or equal to $N - M$ the proposed algorithm yields the exact solution for block transforms as derived in [8]. This can be seen from the properties of the correlation matrices $\boldsymbol{\mathcal{R}}(k)$. Because the channel matrix $\boldsymbol{C}(z)$ reduces to $\boldsymbol{C}(z) = \boldsymbol{C}_0 + z^{-1}\boldsymbol{C}_1$ the matrices $\boldsymbol{\mathcal{R}}(k)$ are nonzero only for $k = -1, 0, 1$. $\boldsymbol{\mathcal{R}}(0)$ has rank N whereas $\boldsymbol{\mathcal{R}}(-1)$ and $\boldsymbol{\mathcal{R}}(1)$ only have rank L. Thus, the proposed algorithm will lead to $\boldsymbol{P}^H\boldsymbol{\mathcal{R}}(k)\boldsymbol{P} = \mathbf{0}$ for $k \neq 0$ which means that all IBI will be canceled. The form of $\boldsymbol{P}$ can be seen from the fact that $\boldsymbol{C}_1$ is nonzero only in the first L rows. Therefore, the algorithm yields

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{0}_{L\times(N-L)} \\ \mathbf{0}_{(N-L)\times L} & \boldsymbol{I}_{(N-L)\times(N-L)} \end{bmatrix},$$

$$\boldsymbol{P}\boldsymbol{G}_0 = \boldsymbol{G}_0 = \begin{bmatrix} \mathbf{0}_{L\times M} \\ \boldsymbol{G}_{(N-L)\times M} \end{bmatrix}$$

with some matrix $\boldsymbol{G}_{(N-L)\times M}$. This structure of $\boldsymbol{G}_0$ corresponds to the leading zero method of [8].

IV. EXAMPLES

The first example considers a configuration where the channel order L is considerably smaller than the number of subchannels. The chosen parameters are $L = 4$ and $N = 16$, and the E_b/N_0 ratio at the receiver input is set to $30dB$. The channel impulse response is $c(n) = [1,\ 1,\ 1,\ 1,\ 1]$. All channel zeros lie on the unit circle of the z-plane. The frequency response of the channel is depicted in Fig. 3. A joint transmitter/receiver design according the proposed algorithm has been carried out for M between 12 and 16. The obtained SNR's at the receiver output are depicted in Fig. 4. One can see that the highest SNR's are obtained for $M = 12$ which is the case where $L = N - M$ and no IBI occurs. The SNR's decrease gradually with an increasing M. The results for $M = 16$ (no redundancy), however, are substantially inferior to the other ones. This shows that already a minimum amount of redundancy may yield a significant performance enhancement over the case where no redundancy is introduced.

In a second example we consider a configuration where the channel order is considerably higher than N. The parameters are $L = 30$, $N = 16$, and $E_b/N_0 = 30dB$. The channel has been designed with the Remez algorithm to be a lowpass filter with large ripple. Its frequency response is depicted in Fig. 5. With $N = 16$ and $L = 30$ the IBI amounts to three blocks at the receiver input. Note that this case cannot be treated with the algorithm of [8], and the algorithm of [10] would lead to IIR transmit filters. Further note that to allow for block transmission without IBI, one would have to increase N substantially. However, this would introduce a large delay if an acceptable ratio M/N was to be maintained. The proposed algorithm, on the other hand, is able to carry out the joint transmitter/receiver design. Fig. 6 shows a comparison of the SNR's obtained with the proposed algorithm and the simple precoding of [7] using

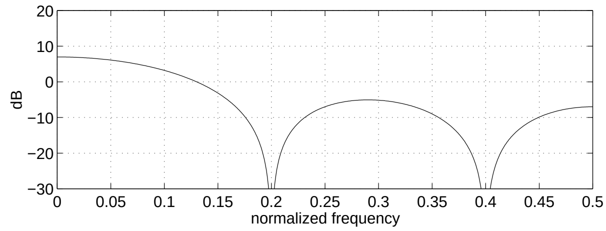

Fig. 3. Frequency response of channel $c(n) = [1,\ 1,\ 1,\ 1,\ 1]$.

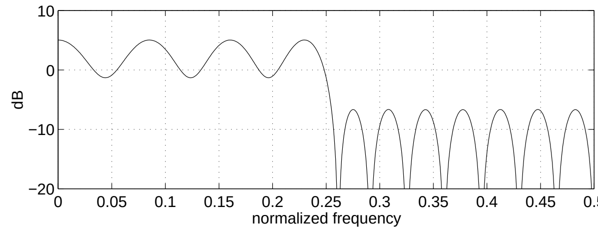

Fig. 5. Frequency response of channel (Remez design).

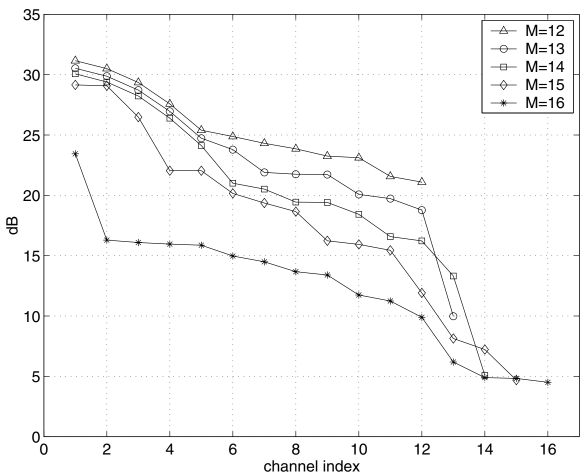

Fig. 4. Signal to noise ratios at receiver outputs. Channel $c(n) = [1,\ 1,\ 1,\ 1,\ 1]$; $N = 16$; $M = 12,\ 13, \ldots,\ 16$; $E_b/N_0 = 30dB$.

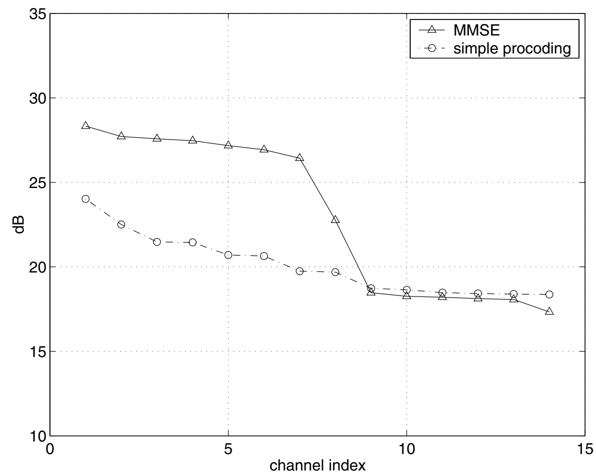

Fig. 6. Signal to noise ratios at receiver outputs for MMSE and simple precoding. $L = 30$; $N = 16$; $M = 14$; $E_b/N_0 = 30dB$.

the transmit matrix $\boldsymbol{G}_0 = [\boldsymbol{I}_{M\times M},\ \mathbf{0}_{(N-M)\times M}]^T$. In both cases the optimal MMSE receivers have been employed. As one can see, the optimized transmitter yields a significant performance enhancement over the simple one.

V. Conclusions

A method for the joint design of transmitter and receiver for data transmission over dispersive channels has been presented. The proposed method can treat the practically important case where the transmitter is FIR and the channel has arbitrary length. This allows for low latency transmission over dispersive channels. Design examples have confirmed the effectiveness of the design method.

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