# Memory Truncation and Crosstalk Cancellation in Transmultiplexers

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***Abstract*—This letter addresses the design of linear networks that reduce intersymbol interference and crosstalk in transmultiplexers. The proposed filter design method is based on the maximization of the signal-to-noise ratio at the detector input, defined for channel memories being truncated to arbitrarily short lengths. Thus, low-complexity Viterbi detectors working independently for all data channels can be used. The design of minimum mean-square error equalizer networks is included in the framework.**

***Index Terms*— Crosstalk, equalizers, intersymbol interference, transmultiplexing.**

## I. Introduction

TRANSMULTIPLEXERS are systems that convert time-division multiplexed signals into frequency-division multiplexed signals and vice versa [1]. Essentially, these systems are filter banks as shown in Fig. 1. The transmission from input $i$ to output $k$ is described by the impulse responses

$$t_{i,k}(m) = q_{i,k}(mN) \tag{1}$$

where $q_{i,k}(n) = g_i(n) * c(n) * h_k(n)$. Herein, the asterisk denotes convolution, $g_i(n)$ is the $i$th synthesis filter, $c(n)$ is the channel, and $h_k(n)$ is the $k$th analysis filter.

In the noise-free case, perfect reconstruction (PR) of the input data with a delay of $m_0$ samples is obtained if the condition

$$t_{i,k}(m) = \delta_{i,k}\delta_{m,m_0}, \qquad i, k = 0, 1, \cdots, M-1 \tag{2}$$

is satisfied. $\delta_{i,k}$ denotes the Kronecker symbol. This condition is met for PR filter banks and an ideal channel $C(z) = 1$. In practice, when having a nonideal channel, at least intersymbol interference will arise. In addition, when a critically sampled system ($N = M$) is used, the frequency bands necessarily overlap and crosstalk between different data channels occurs. Solutions to this problem based on various types of equalizers have been proposed [2]–[4]. However, it is well known that receivers based on maximum likelihood detection via the Viterbi algorithm are superior to those based on linear equalizers [5]. The only drawback of Viterbi detectors is their complexity. For example, in the transmultiplexer case, maximum-likelihood detection requires that all data sequences $d_k(m)$ and all impulse responses $t_{i,k}(m)$ are considered simultaneously in the receiver.

In order to overcome the complexity problem of Viterbi detectors for long channel impulse responses, the concept of memory truncation has been introduced in [6]. This technique can be viewed as an equalization the noncritical part of a transmission channel, leaving the critical part (i.e., the part with zeros close to the unit circle) for a subsequent Viterbi detection. In this paper, the memory truncation approach is extended to the transmultiplexer case. The proposed receiver uses linear filters for both crosstalk cancellation and memory truncation.

*Notation:* $E\{\}$ denotes the expectation operation. The superscript $T$ denotes transposition of a vector or matrix. The superscripts $*$ and $H$ denote complex conjugation and conjugate transposition ($\mathbf{r}^H = [\mathbf{r}^*]^T$), respectively.

## II. Network Design

For the following derivation let us assume that crosstalk only appears between adjacent channels. The equalizer network for the $k$th data channel is shown in Fig. 2. It takes the signals in channels $k-1$, $k$, and $k+1$ into account. The systems $h_{k-1,k}(m)$ and $h_{k+1,k}(m)$ are used for crosstalk cancellation while the system $h_{k,k}(m)$ is the equalizer used for memory truncation. The system with impulse response $p_k(m)$ in Fig. 2 is the residual system, which has to be considered in the Viterbi detector.

The following optimality criterion is used for the filter design:

$$E\{|e_k(m)|^2\} \stackrel{!}{=} \min \tag{3}$$

with

$$\begin{aligned} e_k(m) = {} & r_{k-1}(m) * h_{k-1,k}(m) + r_k(m) * h_{k,k}(m) \\ & + r_{k+1}(m) * h_{k+1,k}(m) - d_k(m-m_0) * p_k(m). \end{aligned} \tag{4}$$

The lengths $L_{p_k}$ of the residual impulse responses $p_k(m)$, the lengths $L_{h_{j,k}}$ of the prefilters $h_{j,k}(m)$, and the overall delay $m_0$ are arbitrary. For the sake of notational simplicity, we will assume equal lengths $L_{p_k} := L_p$ and $L_{h_{j,k}} := L_h$ in the following. Note that for the choice $L_p = 1$ the memory truncation approach reduces to an MMSE approach that takes adjacent channels into account.

In order to solve the design problem, $e_k(m)$ according to (4) is written as

$$e_k(m) = \mathbf{r}_k^T(m)\mathbf{h}_k - \mathbf{d}_k^T(m-m_0)\mathbf{p}_k \tag{5}$$

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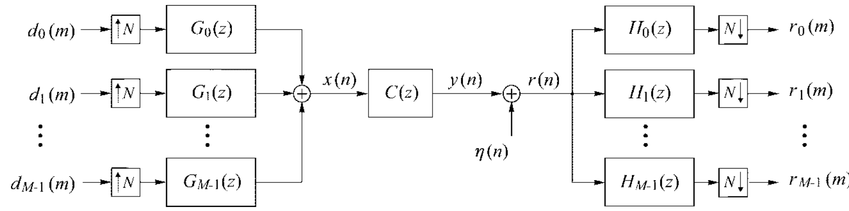


Fig. 1. Transmultiplexer with transmission channel and additive noise.

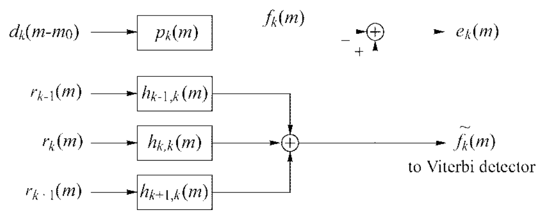


Fig. 2. Filter network for memory truncation and crosstalk cancellation in the $k$th data channel.

where

$$\begin{aligned}
\mathbf{h}_k^T &= [h_{k-1,k}(0), \cdots, h_{k-1,k}(L_h-1), h_{k,k}(0), \cdots, \\
&\quad h_{k,k}(L_h-1), h_{k+1,k}(0), \cdots, h_{k+1,k}(L_h-1)], \\
\mathbf{r}_k^T(m) &= [r_{k-1}(m), \cdots, r_{k-1}(m-L_h+1), \\
&\quad r_k(m), \cdots, r_k(m-L_h+1), \\
&\quad r_{k+1}(m), \cdots, r_{k+1}(m-L_h+1)], \\
\mathbf{p}_k^T &= [p_k(0), \cdots, p_k(L_p-1)], \\
\mathbf{d}_k^T(m) &= [d_k(m), \cdots, d_k(m-L_p+1)].
\end{aligned}$$

Assuming stationary data processes $d_k(m)$, $k = 0, 1, \cdots, M-1$, a time invariant channel $c(n)$, and a stationary additive noise process $\eta(n)$, the criterion (3) becomes

$$\mathbf{h}_k^H \mathbf{R}_{rr}^{(k)} \mathbf{h}_k - \mathbf{h}_k^H \mathbf{R}_{rd}^{(k)} \mathbf{p}_k - \mathbf{p}_k^H \mathbf{R}_{dr}^{(k)} \mathbf{h}_k + \mathbf{p}_k^H \mathbf{R}_{dd}^{(k)} \mathbf{p}_k \overset{!}{=} \min \quad (6)$$

where

$$\begin{aligned}
\mathbf{R}_{rr}^{(k)} &= E\{\mathbf{r}_k^*(m)\mathbf{r}_k^T(m)\} \\
\mathbf{R}_{rd}^{(k)} &= [\mathbf{R}_{dr}^{(k)}]^H = E\{\mathbf{r}_k^*(m)\mathbf{d}_k^T(m-m_0)\} \\
\mathbf{R}_{dd}^{(k)} &= E\{\mathbf{d}_k^*(m-m_0)\mathbf{d}_k^T(m-m_0)\}.
\end{aligned}$$

In the following, uncorrelated data are assumed: $\mathbf{R}_{dd} = \sigma_d^2 \mathbf{I}$. As proposed in [6] for the single-channel case, we first find the optimal vector $\mathbf{h}_k$ for a fixed vector $\mathbf{p}_k$ in the sense of (6) as $\mathbf{h}_{opt} = \mathbf{R}_{rr}^{-1}\mathbf{R}_{rd}\mathbf{p}$.[1] Substitution of $\mathbf{h}_{opt}$ into (6) and solving the remaining problem under the condition $\mathbf{p}^H\mathbf{p} = 1$ leads to the following eigenvalue problem:

$$[\sigma_d^2 \mathbf{I} - \mathbf{R}_{dr}^H \mathbf{R}_{rr}^{-1} \mathbf{R}_{rd}]\mathbf{p} = \lambda \mathbf{p}. \quad (7)$$

The optimal vector $\mathbf{p}$ is the eigenvector that belongs to the smallest eigenvalue $\lambda$.

An alternative to the eigenvector solution shown above is the extension of the filter design method in [7] to the transmultiplexer case. Here we solve (6) under the condition $p(0) = 1$. With $\mathbf{p}^T = [1, \tilde{\mathbf{p}}^T]$, the criterion (6) leads to the following linear set of equations for $\tilde{\mathbf{p}}$ and $\mathbf{h}$:

$$\begin{bmatrix} \tilde{\mathbf{R}}_{dd} & -\tilde{\mathbf{R}}_{rd}^H \\ -\tilde{\mathbf{R}}_{rd} & \mathbf{R}_{rr} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{p}} \\ \mathbf{h} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{r}_{rd} \end{bmatrix} \quad (8)$$

where

$$\mathbf{R}_{dd} = \begin{bmatrix} \sigma_d^2 & \mathbf{0}^T \\ \mathbf{0} & \tilde{\mathbf{R}}_{dd} \end{bmatrix}, \quad \mathbf{R}_{rd} = [\mathbf{r}_{rd}, \tilde{\mathbf{R}}_{rd}]. \quad (9)$$

Note that (8) can be solved efficiently via partitioned inversion or simply by adaptation.

Although the solution to (8) yields filters that are optimal in the sense of (6) with respect to $p(0) = 1$, a slight modification may be useful. In order to explain this modification, let us first

[1] In order to simplify the notation, the subscript $k$ and the superscript $(k)$ are omitted.

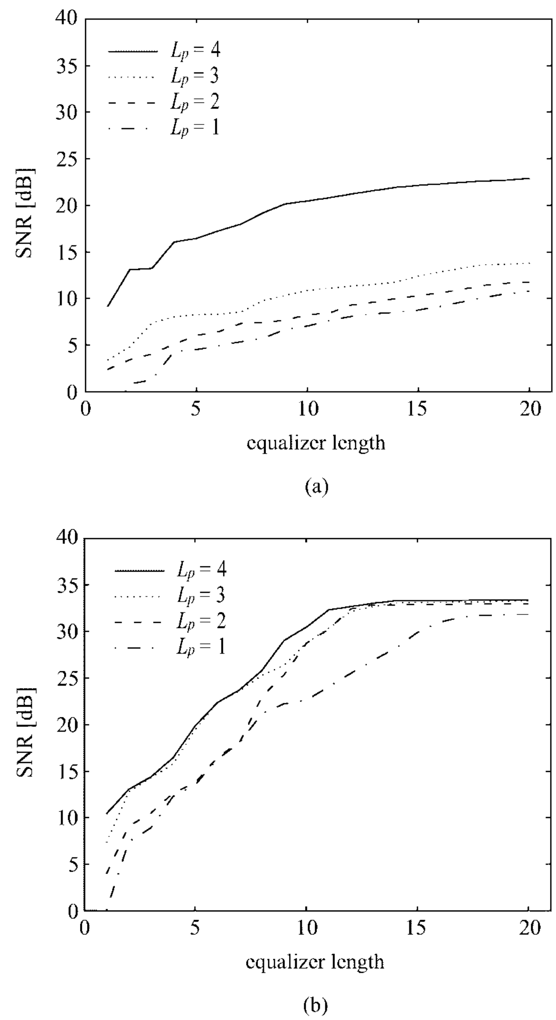


Fig. 3. SNR's in second and fourth data channel.

observe that the upper part of (8) gives

$$\sigma_d^2 \tilde{\mathbf{p}}_k = [\tilde{\mathbf{R}}_{rd}^{(k)}]^H \mathbf{h}_k. \tag{10}$$

This means that the coefficients in $\tilde{\mathbf{p}}_k$ are equal to $L_p - 1$ of the $L_p$ dominant consecutive values of the overall impulse response $\tilde{t}_{k,k}(n)$ which describes the relationship between the input signal $d_k(m)$ and the output signal $\tilde{d}_k(m)$:

$$\tilde{t}_{k,k}(m) = t_{k,k-1}(m) * h_{k-1,k}(m) + t_{k,k}(m) * h_{k,k}(m) + t_{k,k+1}(m) * h_{k+1,k}(m). \tag{11}$$

If we want the impulse response $p_k(m)$ to be equal to all $L_p$ dominant values of $\tilde{t}_{k,k}(n)$, we have to recompute $p_k(0)$. From the restriction $\sigma_d^2 \mathbf{p}_k = [\mathbf{R}_{rd}^{(k)}]^H \mathbf{h}_k$ with $\mathbf{p}_k^T = [p_k(0), \tilde{\mathbf{p}}_k^T]$, we get

$$p_k(0) = \frac{1}{\sigma_d^2} [\mathbf{r}_{rd}^{(k)}]^H \mathbf{h}_k. \tag{12}$$

Clearly, the recomputation of $p_k(0)$ decreases the SNR, but we have the advantage of an unbiased model, which is easily used in the Viterbi detector.

## III. Example

In this example, a near-PR MDFT filter bank according to [8] with $N = M = 8$ is used. The prototype is a raised-cosine filter with roll-off factor 0.5 and a stopband attenuation of 40 dB. Fig. 3 shows the SNR's for the second and fourth data channel for different lengths $L_p$ and $L_h$ and a channel with impulse response $c(n) = \{1, -0.2, 0.82\}$. The SNR is defined as $\mathrm{SNR} = E\{|f_k(m)|^2\}/E\{|e_k(m)|^2\}$ with $f_k(m) = d_k(m-m_0) * p_k(m)$. The SNR at the receiver input, defined as $\mathrm{SNR}_0 = E\{|y(n)|^2\}/E\{|\eta(n)|^2\}$, is 30 dB in this example. The results for channel 2, the worst channel (together with channel 6), show that the SNR can be dramatically increased when going from MMSE equalization ($L_p = 1$) to memory truncation ($L_p > 1$). For channel 4, the best of all eight channels, convergence to the same SNR for all $L_p$ can be observed.

## IV. Conclusion

In this letter, new receiver concepts for data transmission with transmultiplexers based on memory truncation were presented. The results show that memory truncation leads to remarkably higher SNR's than MMSE equalization. Therefore, a very good performance (even in the case of critical channels) can be expected from this method. The network can be easily extended to the case where crosstalk appears between all channels.

## References


[1] M. Vetterli, "Perfect transmultiplexers," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Tokyo, Japan, Apr. 1986, pp. 48.9.1–48.9.4.

[2] R. P. Ramachandran and P. Kabal, "Transmultiplexers: Perfect reconstruction and compensation of channel distortion," *Signal Processing,* vol. 21, no. 3, pp. 261–274, Nov. 1990.

[3] N. Fliege and G. Rösel, "Equalizers and crosstalk compensation filters for DFT polyphase transmultiplexer filter banks," in *Proc. IEEE Int. Symp. on Circuits and Systems,* London, U.K., 1994, vol. III, pp. 173–176.

[4] T. Wiegand and N. J. Fleige, "Equalizers for transmultiplexers in orthonogal multiple carrier data transmission," *Annals Telecommun.*, vol. 52, no. 1–2, pp. 39–45, Jan. 1997.

[5] G. D. Forney, "Maximum likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inform. Theory,* vol. IT-18, pp. 363–378, May 1972.

[6] D. D. Falconer and F. R. Magee, "Adaptive channel memory truncation for maximum likelihood sequence estimation," *Bell Syst. Tech. J.*, vol. 52, no. 9, pp. 1541–1562, 1973.

[7] K. D. Kammeyer, "Time truncation of channel impulse responses by linear filtering: A method to reduce the complexity of Viterbi equalization," *Arch. Elektrotech. Übertragung.*, vol. 48, no. 5, pp. 237–243, 1994.

[8] N. J. Fliege, *Multirate Digital Signal Processing.* Chichester, U.K.: Wiley, 1994.