Joint Multiresolution Magnetic Particle Imaging and System Matrix Compression

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I. Introduction

Magnetic particle imaging (MPI) is a tracer-based medical imaging method that is based on the nonlinear magnetization characteristics of super-paramagnetic iron oxide nanoparticles (SPIOs) [1]. With different accelerated and static magnetic fields, the MPI-scanner generates a small area in which the magnetic fields neutralize each other. The area is called the field free point (FFP). The FFP is normally periodically moved on a pre-defined trajectory over the whole field of view (FOV). The change of magnetization leads to an induced voltage in a receive coil, where, due to the nonlinear magnetization characteristics of the SPIOs, only SPIOs from the vicinity of the FFP contribute significantly to the measured signal. For MPI-scanners with a Lissajous FFP-trajectory, the system response normally has to be measured. For this, a probe of SPIOs material is placed on different spatial positions, and the responses are saved in a so-called system matrix. With help of the system matrix, the inverse problem of estimating the SPIOs' distribution from the voltage signal can be solved. Unfortunately, the system matrix can be very dense and huge in size. For a dense matrix, the reconstruction process can be very slow. In [2], it was observed that the system matrix of MPI-scanners with a FFP traveling along a Lissajous-trajectory can be highly compressed by the discrete cosine transform (DCT) followed by thresholding. Recently, a work for matrix compression was published on a non-Euclidian grid, where the Chebyshev transform becomes orthogonal and the compression performance is even improved [3]. In this work, we develop a multiresolution representation for the system matrix. In particular, we use a combination of the DCT-II and the discrete wavelet transform (DWT) for the joint system-matrix compression and multiresolution reconstruction.

II. Material and Methods

The reconstruction problem in MPI can be described as a linear inverse problem by

$$Sc \approx f \,, \tag{1}$$

where $S \in \mathbb{C}^{M\times N}$ denotes the system matrix, $c \in \mathbb{R}_+^N$ is the positive unknown particle distribution, and $f \in \mathbb{C}^M$ contains the frequency components of the voltage signal. Following the work in [2], the reconstruction problem in (1) can also be expressed in a transform domain as

$$S_T c_T = STT^{-1}c \approx f \,, \tag{2}$$

where $T \in \mathbb{R}^{N\times N}$ describes an invertible transform, S_T is the system matrix in the transform domain, and c_T the particle distribution in the transform domain. The idea is to choose T in such a way that the matrix $S_T = ST$ has many small components that become zero after thresholding. It is well known that the DCT-I and -II are such kinds of transform for MPI system matrices. Unfortunately, the DCT is a global transform on the spatial domain and offers no strategy for a multiresolution analysis (MRA). One transform to apply a MRA is the DWT [4], which can also be represented as a linear transform T. Nevertheless, the DWT is not as sparsifying as the DCT for MPI system matrices. The good news is that the lowpass filtered and downsampled coefficients of the system matrix can be interpreted as a coarser version of the system matrix. By combining the advantages of both transforms, we develop an MRA for MPI system matrices. The system matrix will be decomposed level-wise by the d-dimensional DWT with respect to the dimensionality $d \in \{1,2,3\}$ of the particle distribution, followed by a DCT of the lowpass filtered coefficients of the system matrix. For demonstration purposes, a two-dimensional decomposition is shown in Fig. 1. For each stage of the two-dimensional DWT, the system matrix

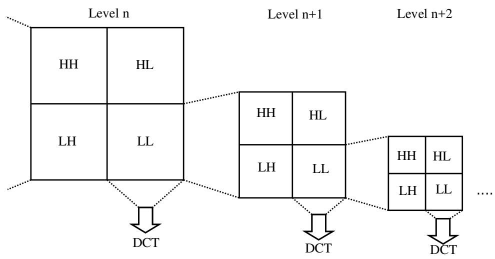

***Figure 1**: The spatial dimension of the system matrix is decomposed by the discrete wavelet transform to form a multiresolution pyramid. In HH both dimensions are highpass filtered, in LH/HL one of the dimensions is highpass and the other is lowpass filtered, and in LL both dimensions are lowpass filtered.*

will be decomposed into four submatrices. The lowpass filtered version of the system matrix will then be transformed with the DCT to the compressive domain and is also the basis for the next level of the decomposition to form a spatial pyramid. The formulation is independent of the used reconstruction method. Thus, the introduced formulation of the system matrix can be included in different solvers. We considered the Tikhonov regularized least square reconstruction problem

$$c^{\ell} = \arg\min_{c\in\mathbb{R}_{+}^{K_\ell}} \left\|S_T^{\ell} T_\ell^{-1} c - f\right\|_2^2 + \lambda^2 \|c\|_2^2 \quad (3)$$

with $S_T^\ell \in \mathbb{C}^{M\times K_\ell}$ being the compressed system matrix on the decomposition stage ℓ, $\lambda > 0$, and $T_\ell^{-1} \in \mathbb{R}^{K_\ell \times K_\ell}$ representing the DWT+DCT transform of the ℓ-level, where $K_0 = N = N_x N_y N_z$ denotes the number of voxels of the particle distribution. The number of voxels of each subspace problem in (3) is $K_\ell = \left\lceil \frac{N_x}{2^\ell} \right\rceil \left\lceil \frac{N_y}{2^\ell} \right\rceil \left\lceil \frac{N_z}{2^\ell} \right\rceil$ where $\lceil\cdot\rceil$ means the ceiling operator. We started the reconstruction on the coarse level L_{max} and ended on the finest level 0. The SPIOs distribution from the coarser resolution will be used as input for the next finer resolution stage by inserting it into the low-resolution components of the finer stage and the unknown high-resolution components initialized with zeros for the iterative reconstruction. The reconstruction problem is solved by a variation of the fast iterative shrinkage-thresholding [5], where we replaced the ℓ_1-constraint by a non-negativity and an ℓ_2-constraint. We tested the approach on the simulated Lissajous-trajectory MPI system matrix dataset which was also used in [6]. For the simulated dataset, the first 62 frequency components were deleted. To speed up the reconstruction, the system matrix was globally thresholded, so that every sublevel system matrix retained 99.7% of its energy. The DWT was implemented in a non-expansive form with biorthogonal 9/7-wavelets for arbitrary signal length in lifting structure [4]. We reconstructed the SPIOs distribution with ($\lambda = 0.35$) and without ($\lambda = 10^{-4}$) energy normalization of the rows of the subsystem matrix. Both regularization parameters were selected by hand. As stopping criterion, an upper limit for the relative change of the objective function was set.

III. Results

Due to the limited space, we exemplarily show only one example of SPIOs-distribution reconstruction on different levels in Fig. 2. It can be recognized that the structure of the 250x250-pixel distribution is also identifiable inside the low-resolution reconstructions. It can be observed that the 63x63-pixel reconstruction still has a good separation between the particle spots and that the shapes are quite similar to the 250x250 high-resolution version. The 32x32-pixel reconstruction, however, has significant artifacts and deformations of the particle spots.

IV. Discussion

The presented method offers a combination of two matrix compression techniques and gives rise to an MRA. It has the ability to first reconstruct the particle distribution on a coarse level, and, if more computational power is at hand,

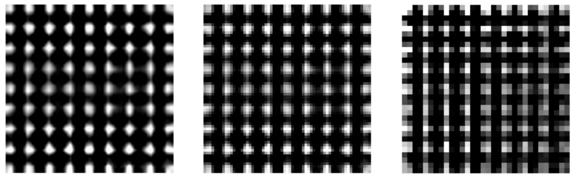

Figure 2*: The SPIOs reconstruction with energy normalization of the MRA from left to right: 250x250, 63x63, and 32x32 pixels. The images are clipped to [0, 2^{level}] for visualization.*

a high-resolution reconstruction can be performed. The compression is not a necessary part of this method. The method can also be used for a fast reconstruction inside a multiresolution analysis without compression of the system matrix. A possible application of the developed method could be to find the support of the SPIOs distribution inside the FOV and to exclude regions without particle distribution earlier on the finer grid, where the linear system's condition is typically worse. In addition, an interesting scenario for the approach with its joint local and global transforms could be the use inside a compressed-sensing framework for the reconstruction of SPIO distributions. Since the DWT and DCT offer compression for both the system matrix and the image to be reconstructed, our developed transform offers us the best of two worlds: A good compressive transform for the system matrix and a good compressive transform for the particle distribution, which is highly promising for this purpose.

V. Conclusions

We developed an MRA formulation for MPI based on the DWT and the DCT-II. We were able to show the efficiency of our approach, which offers the possibility to proceed step-wise from a coarse level to a high-resolution reconstruction of the SPIOs distribution. Our future research will be directed toward developing a compressed sensing based reconstruction of the particle distributions using our MRA formulation of the system matrix.

Acknowledgements

This work was supported by the German Research Foundation under grant number ME 1170/7-1.

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