Soft-Input Reconstruction of Binary Transmitted Quantized Overcomplete Expansions

Jörg Kliewer, *Senior Member, IEEE,* and Alfred Mertins, *Senior Member, IEEE*

***Abstract*—In this letter, we propose a soft-decoding method for quantized overcomplete frame expansions that are binary transmitted through noisy channels. The frame expansions can be viewed as real-valued block codes that are directly applied to waveform signals prior to quantization. The explicit redundancy introduced in the continuous amplitude domain is exploited by the decoder in two stages. First, the index-based redundancy is used by a soft-input soft-output source decoding approach that outputs decoded symbols together with their reliability information. In a second stage, the soft information on the symbols and the structure of the introduced redundancy are used to correct errors. The performance of the proposed approach is evaluated for different code constructions based on the discrete Fourier transform (DFT), the discrete cosine transform (DCT), and the discrete Hadamard transform (DHT), and is compared to standard approaches without soft decoding.**

***Index Terms*—Joint source-channel coding, overcomplete frame expansions, real-valued block codes, residual source redundancy.**

I. Introduction

SHANNON'S source-channel separation principle is the basis for the classical signal transmission strategy, where the signals are first compressed as best as possible and then explicit redundancy is added for error protection. However, in recent years, it has been shown that especially for delay- and complexity-constrained systems a better performance can be achieved with combined source-channel coding or decoding techniques. Some of these approaches keep the classical structure and carry out a joint allocation of source and channel coding rates [1], [2], while others do not use binary channel codes at all and design the source encoder such that the residual index-based redundancy in the resulting bitstream alone is sufficient to provide reasonable error protection [3], [4]. The first class provides excellent results for moderately distorted channels, however, especially for low channel signal-to-noise ratios (SNRs) their performance highly depends on the properties of the used channel codes. The methods in the second class often have less encoding delay and complexity, and for very low channel SNRs, they often yield similar or better performance than the combination of strong source and channel encoding.

Manuscript received March 1, 2004; revised May 4, 2004. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Xiang-Gen Xia.

J. Kliewer is with the University of Southampton, Communications Research Group, School of Electronics and Computer Science, Southampton SO17 1BJ, U.K., on leave from Institute for Circuits and Systems Theory, University of Kiel, 24143 Kiel, Germany (e-mail: j.kliewer@soton.ac.uk).

A. Mertins is with the Signal Processing Group, Institute of Physics, University of Oldenburg, 26111 Oldenburg, Germany (e-mail: alfred.mertins@uni-oldenburg.de).

Digital Object Identifier 10.1109/LSP.2004.836941

On the other hand, overcomplete signal expansions, where the redundancy for error protection is inserted prior to the quantization stage of the source encoder, have been recently suggested as an alternative to classical forward error protection (FEC) approaches [5]–[10]. In this letter, we follow the idea of [7] and insert explicit redundancy by applying structured overcomplete signal expansions to nonoverlapping blocks of input samples, resulting in real-valued block channel codes. For the code design, we study construction principles based on the discrete Fourier transform (DFT) leading to real-valued BCH codes [11], the discrete cosine transform (DCT), and the discrete Hadamard transform (DHT). The novelty of our approach lies mainly in the decoding process, which is carried out in the following stages: First, the unequal symbol transition probabilities that are present due to the overcomplete signal representation are utilized for soft-input soft-output (SISO) source decoding. Then, the structure of the introduced redundancy is exploited by using the reliability information that is available from the previous decoding stage. Finally, the waveform signal is reconstructed through the application of the pseudo-inverse of the overcomplete block transform. The performance of the proposed combined source–channel coding and decoding approach is studied for signal transmission over AWGN channels.

II. Transmission System

The block diagram of the overall transmission system is depicted in Fig. 1. The real-valued symbols $U_k \in \mathbb{R}$ represent samples of a correlated source signal, where we assume that the source correlation can be described as a first-order autoregressive process (AR(1)), which represents a good correlation model for many waveform source signals. First, the symbols U_k are grouped into nonoverlapping blocks of K symbols, and then for each block an overcomplete frame expansion with the frame operator $\mathbf{G}$ of dimension $N \times K$ with $N > K$ is carried out. Similar to [5], the obtained symbols are quantized with M-bit quantizers and transmitted. The redundancy introduced prior to quantization has two effects. First, it gives us some information on the original symbols U_k, which can be utilized in the decoder to reduce the quantization effects irrespective of any transmission errors. Second, in analogy with the theory of real-valued BCH-codes [11], the matrix $\mathbf{G}$ can be interpreted as a generator matrix of the underlying channel code with a code rate of $R = K/N$.

In accordance with [7] and [8], the matrix $\mathbf{G}$ is defined as

$$\mathbf{G} = \sqrt{\frac{N}{K}}\mathbf{T}_N^H \mathbf{P} \mathbf{T}_K. \tag{1}$$

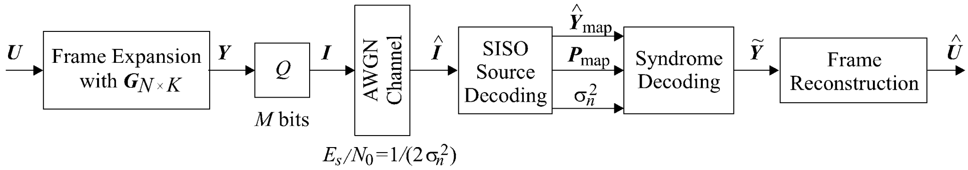

Fig. 1. Model of the transmission system.

Herein, T_N and T_K denote unitary transform matrices of size $N \times N$ and $K \times K$, respectively. The matrix $\mathbf{P} \in \mathbb{R}^{N \times K}$ has nonzero elements only on two diagonals, denoted in the following by $\mathbf{d}_u$ and $\mathbf{d}_l$, respectively, and serves to introduce redundancy into the data sequences. The term $\mathbf{d}_u$ is the upper diagonal starting in the upper left corner of $\mathbf{P}$, and $\mathbf{d}_l$ is the lower diagonal that ends in the lower right corner of $\mathbf{P}$. For the most interesting case, where the DFT is used as underlying transform, the definitions depend on K being even or odd. For even K, the upper and lower diagonal vectors are given by

$$\mathbf{d}_u = \left[\underbrace{1,\ldots,1}_{K/2 \text{ elem.}}, \frac{1}{\sqrt{2}}, 0, \ldots, 0\right],$$

$$\mathbf{d}_l = \left[\underbrace{0,\ldots,0}_{K/2 \text{ elem.}}, \frac{1}{\sqrt{2}}, 1, \ldots, 1\right].$$

For odd K, we have

$$\mathbf{d}_u = \left[\underbrace{1,\ldots,1}_{(K+1)/2 \text{ elem.}}, 0, \ldots, 0\right],$$

$$\mathbf{d}_l = \left[\underbrace{0,\ldots,0}_{(K+1)/2 \text{ elem.}}, 1, \ldots, 1\right].$$

If K is odd, then $\mathbf{P}$ introduces $N-K$ consecutive zeros in the center of the coefficient vector. For even K, a total number of $N-K-1$ zeros is introduced, and one coefficient is scaled by the factor $1/\sqrt{2}$ and repeated. Therefore, for both even and odd K, the number of redundant symbols is given by $L = N - K$. The transform using $\mathbf{T}_N$ serves to distribute the introduced redundancy to all N output symbols of a block, such that additional correlation is inserted between all symbols Y_k. Note that for real-valued transforms such as the DCT-II or DHT one could alternatively insert all L zeros at the end of the coefficient vector. However, tests have shown that this does not lead to significant advantages.

The symbol vector $\boldsymbol{Y}$ is scalar quantized with M-bit quantizers, where we obtain the index vector $\boldsymbol{I} = [I_0, I_1, \ldots, I_k, \ldots]$ with $I_k \in \mathcal{I}$, $\mathcal{I} = \{0, 1, \ldots, 2^M - 1\}$. $\boldsymbol{I}$ may also be interpreted as a binary sequence $\boldsymbol{I}_{\text{bin}} = [i_{0,1}, i_{0,2}, \ldots, i_{k,\ell}, \ldots]$ with $i_{k,\ell} \in \{0,1\}$ denoting the ℓth bit of the index I_k. Due to nonperfect source encoding by scalar quantization the source indices I_k show mutual dependencies. For the sake of simplicity these dependencies will be modeled as a first-order stationary Gauss–Markov process with transition probabilities $P(I_k = \lambda | I_{k-1} = \mu)$ where $\mu, \lambda \in \mathcal{I}$. The motivation for an order-one model is that on the one hand a higher model order would strongly increase the decoding complexity of the soft-input soft-output source decoder discussed in Section III, and that on the other hand, a first-order Gauss–Markov process represents a good model for a quantized correlated input process.

The sequence $\boldsymbol{I}$ is transmitted over an AWGN channel with noise variance $\sigma_n^2 = N_0/(2E_s)$, where coherently detected binary-phase shift keying is assumed for the modulation. N_0 denotes the one-sided power spectral density and E_s is the transmit energy per codebit. Then the conditional p.d.f. of a received soft-bit $\hat{i}_{k,\ell}$ is Gaussian-distributed and can be written as

$$p(\hat{i}_{k,\ell} | i_{k,\ell}) = \frac{1}{\sqrt{2\pi}\sigma_n} \cdot \exp\left(-\frac{1}{2\sigma_n^2}(\hat{i}_{k,\ell} - \bar{i}_{k,\ell})^2\right) \qquad (2)$$

with $\bar{i}_{k,\ell} = 1 - 2 \cdot i_{k,\ell}$.

III. Decoder Structure

At the decoder, first a soft-input soft-output (SISO) source decoder is applied to the received soft-bit vector $\hat{\boldsymbol{I}}$. The SISO decoder outputs reliability information for the source hypotheses $I_k = \lambda$ in form of *a posteriori* probabilities (APPs) where an index-based version of the classical BCJR algorithm [12] with the states $I_k = \lambda$ is used as SISO decoder for generating the APPs $P(I_k = \lambda | \hat{\boldsymbol{I}})$. Because for the full BCJR algorithm, in principle, the whole soft-bit vector $\hat{\boldsymbol{I}}$ must have been received before the decoding operation can be started, we mainly restrict ourselves to the forward recursion of the BCJR algorithm in order to keep the system latency low. By just considering the forward recursion we obtain the APPs $P(I_k = \lambda | \hat{\boldsymbol{I}}_0^k)$ which are only conditioned on the received soft-bit vector $\hat{\boldsymbol{I}}_0^k = [\hat{i}_{0,1}, \hat{i}_{0,2}, \ldots, \hat{i}_{k,M}]$ up to the time instant k. Since these APPs can now be generated instantaneously for every k, the system latency remains to be $K-1$ samples as for the case without SISO source decoding. By additionally considering the *a priori* index correlation $P(I_k = \lambda | I_{k-1} = \mu)$, the decoding rule writes

$$P(I_k = \lambda | \hat{\boldsymbol{I}}_0^k) = c_k p(\hat{I}_k | I_k = \lambda) \cdot \sum_{\mu=0}^{2^M-1} P(I_k = \lambda | I_{k-1} = \mu) P(I_{k-1} = \mu | \hat{\boldsymbol{I}}_0^{k-1}) \qquad (3)$$

for $\lambda = 0, 1, \ldots, 2^M - 1$ and the normalization constant c_k. Since the considered AWGN channel is memoryless the channel term $p(\hat{I}_k \mid I_k)$ can be obtained from (2) according to

$$p(\hat{I}_k | I_k) = \prod_{\ell=1}^{M} p(\hat{i}_{k,\ell} | i_{k,\ell}). \qquad (4)$$

Using the APPs from (3), we perform the following maximum *a posteriori* probability (MAP) decoding at the decoder output:

$$\lambda_{\text{map}} = \arg\max_{\lambda} P(I_k = \lambda | \hat{\boldsymbol{I}}_0^k).$$

Given λ_{map}, the corresponding element $\hat{Y}_k$ of the sequence $\hat{\boldsymbol{Y}}$ in Fig. 1 is obtained according to $\hat{Y}_k = y_\lambda$, where y_λ denotes the quantizer reconstruction level corresponding to the index λ_{map}. The corresponding APP $P(I_k = \lambda_{\text{map}} | \hat{\boldsymbol{I}}_0^k)$ represents an element $P_{\text{map},k}$ of the probability sequence $\boldsymbol{P}_{\text{map}}$.

For the further symbol reconstruction, we first construct a matrix $\mathbf{T}_{L\times N}$ which plays the role of a parity check matrix. For odd K, $\mathbf{T}_{L\times N}$ is the submatrix of $\mathbf{T}_N$ that corresponds to the L inserted zeros, such that

$$\mathbf{T}_{L\times N} \cdot \mathbf{y}_N = \mathbf{0}_L \tag{5}$$

where $\mathbf{y}_N$ is a length-N block of the original symbol sequence, and $\mathbf{0}_L$ is a vector of L zeros. For even K, the first $L-1$ rows of $\mathbf{T}_{L\times N}$ consist of the $L-1$ rows of $\mathbf{T}_N$ that correspond to the inserted zeros. The Lth row is the difference of the two rows of $\mathbf{T}_N$ that correspond to the repeated symbol, so that (5) is again satisfied.

Now we consider the vector

$$\mathbf{s} = \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N \tag{6}$$

where $\hat{\mathbf{y}}_N$ denotes a block of N elements taken from $\hat{\boldsymbol{Y}}$. The vector $\mathbf{s}$ is called the *syndrome* and has a similar function as the syndrome in classical BCH codes over finite fields. However, unlike in the binary case, in the method proposed in this letter the norm of the vector $\mathbf{s} = \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N$ does, in general, not completely vanish if there is error-free transmission due to the quantization error introduced after frame expansion.

For the decoding process, we define the corrected symbols as

$$\tilde{\mathbf{y}} = \hat{\mathbf{y}}_N + \hat{\mathbf{e}} \tag{7}$$

and demand that

$$\mathbf{T}_{L\times N} \cdot \tilde{\mathbf{y}} = \mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}] = \mathbf{0}_L. \tag{8}$$

The question is then how to find the correction term $\hat{\mathbf{e}}$ that leads to the best estimate $\tilde{\mathbf{y}}$ for the true vector $\mathbf{y}_N$. Because of $N > L$ the linear system (8) is underdetermined and has infinitely many solutions for $\hat{\mathbf{e}}$.

Estimating $\hat{\mathbf{e}}$ in an optimal way from the information that is available after MAP decoding means to find the vector $\hat{\mathbf{e}}$ which maximizes the conditional p.d.f. $p(\hat{\mathbf{e}}|\sigma_n^2, \mathbf{P}_{\text{map}}, \hat{\mathbf{y}}_N)$ subject to (8). This density will, in general, be non-Gaussian. However, for the reason of simplicity and in order to obtain a linear estimator, we approximate this p.d.f. by a Gaussian distribution. Moreover, we assume that the p.d.f. is independent of $\hat{\mathbf{y}}_N$ and that all components $\hat{e}_i, i = 1, 2, \ldots, N$, of $\hat{\mathbf{e}}$ have zero mean and are mutually uncorrelated. The variances of the components $\hat{e}_i$, denoted by λ_i, are modeled to be dependent on the reliability information $P_{\text{map},i}$ and the normalized channel noise variance σ_n^2. Under the assumptions made above, the conditional p.d.f. of the correction term $\hat{\mathbf{e}}$ is given by

$$p(\hat{\mathbf{e}}|\sigma_n^2, \mathbf{P}_{\text{map}}) = \left((2\pi)^N \prod_{i=1}^{N} \lambda_i\right)^{-1/2} \exp\left(-\frac{1}{2}\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}}\right) \tag{9}$$

where $\boldsymbol{\Lambda} = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_N]$. The quantities λ_i, $i = 1, 2, \ldots, N$, denote the variances of the errors $e_i = \hat{y}_{N,i} - y_{N,i}$ and are approximated as $\tilde{\lambda}_i = \tilde{\lambda}_i(P_{\text{map},i}, \sigma_n^2)$ in the following.

The relationship between $\tilde{\lambda}_i$ and the terms $P_{\text{map},i}$ and σ_n^2 was found experimentally and approximated by a polynomial of the form

$$\tilde{\lambda}_i(P_{\text{map},i}, \sigma_n^2) = \sum_{k=0}^{\kappa} \alpha_k P_{\text{map},i}^k \tag{10}$$

where the coefficients α_k depend on σ_n^2. To determine the coefficients α_k, trials were made where the mean-squared error $E\{|\hat{y}_{N,i} - y_{N,i}|^2\}$ was collected for $P_{\text{map},i}$ lying in intervals $[(q-1)/Q,\ q/Q]$, $q = 1, 2, \ldots, Q$ with $Q = 20$ and for various fixed values of σ_n^2. In a second step, a least-squares polynomial fit of the collected data was carried out to determine the coefficient sets α_k, $k = 0, 1, \ldots, \kappa$, for each value of σ_n^2, where an order $\kappa = 8$ for the approximation polynomial was chosen as a good compromise between complexity and performance.

Maximizing (9) through the choice of $\hat{\mathbf{e}}$ is equivalent to minimizing $\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}}$, so that we obtain the following optimization problem:

$$\text{minimize} \quad \hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}} \quad \text{subject to} \quad \mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}] = \mathbf{0}_L. \tag{11}$$

The solution can be found via the Lagrange multiplier method. We then have to minimize

$$\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}} + \boldsymbol{\mu}^T (\mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}]) \tag{12}$$

through the choice of $\hat{\mathbf{e}}$ and vector $\boldsymbol{\mu}$. The solution to this problem is given by

$$\begin{bmatrix} \hat{\mathbf{e}} \\ \boldsymbol{\mu} \end{bmatrix} = -\begin{bmatrix} 2\boldsymbol{\Lambda}^{-1} & \mathbf{T}_{L\times N}^H \\ \mathbf{T}_{L\times N} & \mathbf{0}_{L\times L} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N \end{bmatrix} \tag{13}$$

and after partitioned inversion, we obtain

$$\hat{\mathbf{e}} = -\boldsymbol{\Lambda} \mathbf{T}_{L\times N}^H [\mathbf{T}_{L\times N} \boldsymbol{\Lambda} \mathbf{T}_{L\times N}^H]^{-1} \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N. \tag{14}$$

Given $\hat{\mathbf{e}}$, we determine $\tilde{\mathbf{y}}$ from (7). A reconstructed block $\hat{\mathbf{u}}$ of the sequence $\hat{\boldsymbol{U}}$ at the output of the reconstruction stage is finally obtained as

$$\hat{\mathbf{u}} = \mathbf{G}^\dagger \tilde{\mathbf{y}} \tag{15}$$

where $\mathbf{G}^\dagger$ denotes the pseudo-inverse of $\mathbf{G}$ and is given by $\mathbf{G}^\dagger = \sqrt{K/N} \mathbf{T}_K^H \mathbf{P}^T \mathbf{T}_N$.

It is worth to mention that the correction term $\hat{\mathbf{e}}$ only has an influence on the final output $\hat{\mathbf{u}}$ when $\boldsymbol{\Lambda}$ is not a multiple of the identity matrix, because due to the orthogonality of $\mathbf{T}_N$ and the given construction of $\mathbf{P}$, we have $\mathbf{G}^\dagger \mathbf{T}_{L\times N}^H = \mathbf{0}_{K\times L}$. Thus, if all received symbols are equally reliable, resulting in equal values λ_i, then no correction will be made.

In terms of decoding complexity the proposed syndrome decoding step has roughly the same order of complexity as the real-valued BCH decoding technique described in [7]. It can be observed from (3) that the BCJR forward recursion leads to approximately 2^{2M} additional multiplications and additions for generating all APPs for one source symbol. The full BCJR decoder requires roughly twice the complexity of the forward recursion [12].

IV. Simulation Results

In order to verify the performance of the resulting transmission system simulations were carried out for an AR(1) input process $\boldsymbol{U}$ with correlation coefficient a and a block length of 48 000 source symbols averaged over 50 simulated AWGN transmissions. The frame expansion uses the parameters $K = 16$ and $N = 32$, and the subsequent scalar uniform quantization has a resolution of $M = 5$ bit. We employ different unitary transforms $\mathbf{T}$, namely, the DFT, the DCT-II, and the DHT, where the forward recursion (3) of the BCJR algorithm and in the DFT-case also the full BCJR is used as SISO source decoder. The performance is compared to an FEC scheme employing a binary $(N', K')_2$ BCH code, which is hard-decoded using the Berlekamp–Massey [13] algorithm and whose parameters are chosen such that approximately the same system latency is achieved as for the proposed approach with the BCJR forward recursion. Furthermore, we compare the proposed decoding method to the (one-dimensional) syndrome decoding approach from [7], where the decoding operation is performed on the hard-decoded AWGN channel outputs. The results are displayed in Figs. 2 and 3 where the reconstruction SNR at the decoder output is plotted over the channel parameter E_b/N_0 with $E_b = E_s/R$. The overall code rate R is given as $R = K/N$ and as $R = K'/N'$ for the FEC scheme, respectively.

We can see from Fig. 2, which shows the result for an uncorrelated input process ($a = 0$), that a strong SNR gain is achieved compared to the method from [7] if SISO source decoding in combination with the proposed frame reconstruction method is used. Clearly, the full BCJR-based source decoder performs best at the expense of a larger system delay. Furthermore, it can be observed that for strongly distorted channels and the clear channel case the proposed transmission technique gives a better performance compared to the FEC-based system with the binary $(127, 64)_2$ BCH code.

Fig. 3 depicts the results for a strongly correlated AR(1) input process with $a = 0.9$. Compared to Fig. 2, here the SNR gain by using additional SISO source decoding is higher due to the source symbol correlation already inherent in the input sequence $\boldsymbol{U}$. In this case, the DCT-II yields a slightly higher SNR than the DFT.

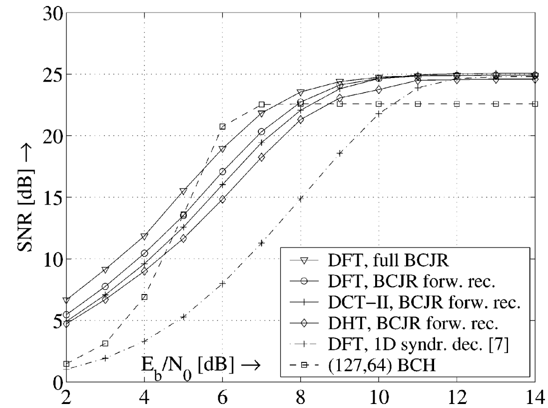

Fig. 2. Reconstruction SNR for an uncorrelated input process (Parameters: $K = 16$, $N = 32$, $M = 5$ bit, 48000 source symbols averaged over 50 simulated AWGN channel transmissions).

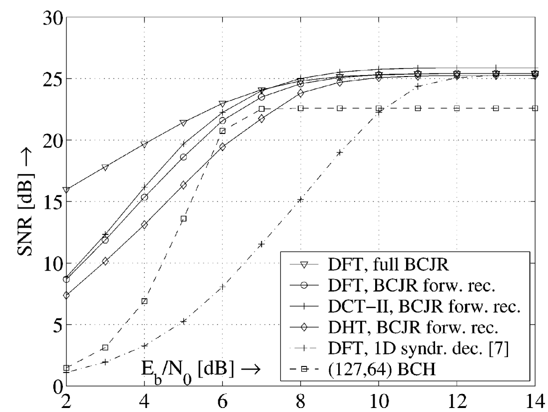

Fig. 3. Reconstruction SNR for a strongly correlated AR(1) input process with $a = 0.9$ (Parameters: $K = 16$, $N = 32$, $M = 5$ bit, 48 000 source symbols averaged over 50 simulated AWGN channel transmissions).

V. Conclusions

We have presented a coding technique that applies real-valued block codes prior to quantization on the encoder side and uses a two-stage soft decoding method on the receiver side. The first decoding stage consists of a SISO source decoder and exploits the index-based redundancy that is present in the transmitted symbols. The second stage is a syndrome decoding method for which we have proposed a dedicated linear estimator for error correction by using the reliability information for the estimated source symbols. Code designs based on the DFT, the DCT-II, and the DHT were studied, of which the best performance was obtained with DFT- and DCT-II-based codes, depending on the source correlation. The soft-decoding strategy proved to be very robust in the presence of noise and a strong performance gain is obtained compared to hard-decoded syndrome decoding of real-valued BCH codes. Furthermore, the presented transmission system generally outperforms classical FEC based on binary BCH codes for both strongly corrupted channels and high channel SNRs where clear channel quality is achieved.

References

[1] B. Hochwald and K. Zeger, "Tradeoff between source and channel coding," *IEEE Trans. Inform. Theory*, vol. 43, no. 5, pp. 1412–1424, Sept. 1997.

[2] P. G. Sherwood and K. Zeger, "Progressive image coding for noisy channels," *IEEE Signal Processing Lett.*, vol. 4, pp. 189–191, July 1997.

[3] K. Sayood and J. C. Borkenhagen, "Use of residual redundancy in the design of joint source/channel coders," *IEEE Trans. Commun.*, vol. 39, no. 6, pp. 838–846, June 1991.

[4] T. Fingscheid and P. Vary, "Robust speech decoding: A universal approach to bit error concealment," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Munich, Germany, Apr. 1997, pp. 1667–1670.

[5] V. K. Goyal, M. Vetterli, and N. T. Thao, "Quantized overcomplete expansions in R^N: Analysis, synthesis, and algorithms," *IEEE Trans. Inform. Theory*, vol. 44, no. 1, pp. 16–31, Jan. 1998.

[6] G. R. Redinbo, "Decoding real block codes: Activity detection, Wiener estimation," *IEEE Trans. Inform. Theory*, vol. 56, no. 2, pp. 609–623, Mar. 2000.

[7] A. Gabay, P. Duhamel, and O. Rioul, "Real BCH codes as joint source channel codes for satellite images coding," in *Proc. IEEE Global Telecommunications Conf.*, San Francisco, CA, 2000, pp. 820–824.

[8] G. Rath, X. Henocq, and C. Guillemot, "Application of DFT codes for robustness to erasures," in *Proc. IEEE Global Telecommunications Conf.*, San Antonio, TX, Nov. 2001, pp. 1246–1250.

[9] F. Labeau, L. Vandendorpe, and B. Macq, "Oversampled filter banks as error correcting codes," in *Proc. 5th Int. Symp. Wireless Personal Multimedia Communications*, vol. 3, Honolulu, HI, Oct. 2002, pp. 1265–1269.

[10] J. C. Chiang, M. Kieffer, and P. Duhamel, "Oversampled filterbanks seen as channel codes: Impulse noise correction," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Hong Kong, Apr. 2003, pp. IV-249–IV-252.

[11] J. K. Wolf, "Redundancy, the discrete Fourier transform, and impulse noise cancellation," *IEEE Trans. Commun.*, vol. COM-31, no. 3, pp. 458–461, Mar. 1983.

[12] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, pp. 284–287, Mar. 1974.

[13] R. E. Blahut, *Algebraic Methods for Signal Processing and Communications Coding*. New York: Springer–Verlag, 1992.