ERROR-RESILIENT TRANSMISSION OF WAVEFORM SIGNALS USING OVERCOMPLETE EXPANSIONS AND SOFT-INPUT SOURCE DECODING

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ABSTRACT

In this paper we present a new approach for robust transmission of waveform signals over noisy channels, where explicit symbol-based redundancy is added to the source signal for error protection by using an overcomplete block transform prior to quantization. This approach is in contrast to commonly used schemes for joint source-channel coding which employ binary channel encoding after the quantization stage. At the decoder, a soft-input soft-output source decoding approach is applied, which allows us to efficiently exploit the explicit redundancy introduced in the continuous amplitude domain. The performance of the proposed approach is evaluated for different code constructions based on the DFT, the DCT, and the discrete Hadamard transform.

1. INTRODUCTION

The classical strategy for efficient signal transmission over noisy channels is to first compress the signals as best as possible using appropriate source coding techniques and then to add explicit redundancy for error protection at the binary level. This is in accordance with Shannon's source-channel separation principle, which states that such systems are asymptotically optimal. In recent years, however, combined source-channel coding techniques and soft-bit decoding algorithms have become an interesting alternative, especially for delay- or complexity constrained systems. One subclass of these approaches is represented by a joint allocation of source and channel coding rates such that the reconstruction error at the decoder is minimized [1]. Often these techniques are used in combination with state-of-the-art source coders and elaborate error protection schemes for the highly sensitive source-encoded bitstreams (e.g. [2, 3]). These methods provide excellent results for moderately distorted channels, however, especially for low channel signal-to-noise ratios (SNRs) their performance highly depends on the properties of the used channel codes. Another subclass of joint source-channel coding is given by joint source-channel decoding, where residual source redundancy is exploited for additional error protection at the decoder. Some approaches even do not use binary channel codes at all and design the source encoder such that the residual index-based redundancy in the resulting bitstream alone is sufficient to provide reasonable error protection at the decoder (e.g. [4, 5]). These methods have less encoding delay and complexity, and for very low channel SNRs, they often yield similar or better performance than the combination of strong source and channel encoding.

In this paper, we follow the idea of [6] and move the introduction of redundancy for error protection prior to the quantization stage of the source encoder. The explicit redundancy is inserted by applying especially structured overcomplete signal expansions to nonoverlapping blocks of input samples, resulting in real-valued block channel codes. For the code design, we study construction principles based on the discrete Fourier transform (DFT) leading to a real-valued BCH code, the discrete cosine transform (DCT), and the discrete Hadamard transform (DHT). The novelty of our approach lies mainly in the decoding process, which is carried out in three stages. First, the source redundancy due to the index correlation introduced by the overcomplete expansion is exploited for soft-bit source decoding, taking the source symbol transition probabilities into account. Then, a real-valued syndrome decoding for the real-valued block code is performed under consideration of the reliability information that is available from the previous source decoding stage. Finally, the waveform signal is reconstructed through the application of the pseudo-inverse of the overcomplete block transform. The performance of the proposed combined source/channel coding and decoding approach is studied for signal transmission over AWGN channels.

2. PROPOSED TRANSMISSION SYSTEM

The block diagram of the overall transmission system is depicted in Figure 1. The vector $\boldsymbol{U} = [U_1, U_2, \ldots, U_k, \ldots]$ represents the real-valued source symbols $U_k \in \mathbb{R}$. In order to obtain an overcomplete frame expansion, similar to [7], nonoverlapping blocks of K symbols from the sequence $\boldsymbol{U}$ are transformed using the frame operator $\mathbf{G}$ of dimension $N \times K$, where $N > K$. In analogy with the theory of real-valued BCH-codes [8], the matrix $\mathbf{G}$ can be interpreted as a generator matrix of the underlying channel code with a code rate of $R_c = K/N$. In accordance with [6, 9], the matrix $\mathbf{G}$ is defined as

$$\mathbf{G} = \sqrt{\frac{N}{K}}\, \mathbf{T}_N^{\mathrm{H}} \mathbf{P}\, \mathbf{T}_K. \tag{1}$$

Herein, T_N and T_K denote unitary transform matrices of size $N \times N$ and $K \times K$, respectively. The matrix $\mathbf{P} \in \mathbb{R}^{N \times K}$ has nonzero elements only on two diagonals, denoted in the following by $\mathbf{d}_u$ and $\mathbf{d}_l$, respectively, and serves to introduce zeros into the data sequences. $\mathbf{d}_u$ is the upper diagonal starting in the upper left corner of $\mathbf{P}$, and $\mathbf{d}_l$ is the lower diagonal that ends in the lower right corner of $\mathbf{P}$. The definitions depend on K being even or odd. For even K, the upper and

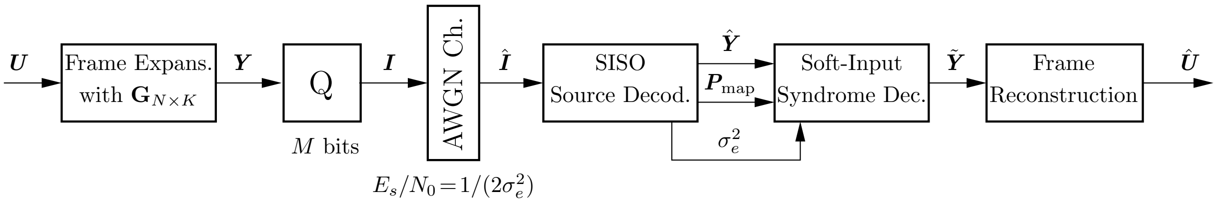

Figure 1: Model of the transmission system

lower diagonal vectors are given by

$$\mathbf{d}_u = [\underbrace{1,\ldots,1}_{K/2 \text{ elements}}, \tfrac{1}{\sqrt{2}}, 0,\ldots,0],$$
$$\mathbf{d}_l = [\underbrace{0,\ldots,0}_{K/2 \text{ elements}}, \tfrac{1}{\sqrt{2}}, 1,\ldots,1].$$

For odd K we have

$$\mathbf{d}_u = [\underbrace{1,\ldots,1}_{(K+1)/2 \text{ elements}}, 0,\ldots,0],$$
$$\mathbf{d}_l = [\underbrace{0,\ldots,0}_{(K+1)/2 \text{ elements}}, 1,\ldots,1].$$

The application of the generator matrix $\mathbf{G}$ in (1) can be viewed as transforming K input symbols into K transform coefficients using transform $\mathbf{T}_K$, inserting zeros into the center of the coefficient vector using $\mathbf{P}$, and carrying out a subsequent transform back to the original domain using $\mathbf{T}_N$ to obtain the symbol vector $\boldsymbol{Y}$. The number of introduced zeros for every K input symbols is given by $N-K-1$ for even K and $N-K$ zeros for odd K, respectively. In addition, for even K, one value is scaled by the factor $1/\sqrt{2}$ and repeated. Thus, for both even and odd K, the number of redundant symbols is given by $L = N-K$.

The vector $\boldsymbol{Y}$ is scalar quantized with M-bit quantizers, where we obtain the index vector $\boldsymbol{I} = [I_1, I_2, \ldots, I_k, \ldots]$ with $I_k \in \mathscr{I}$, $\mathscr{I} = \{0,1,\ldots,2^M-1\}$. $\boldsymbol{I}$ may also be interpreted as a binary sequence $\boldsymbol{I}_{\text{bin}} = [i_{1,1}, i_{1,2}, \ldots, i_{k,\ell}, \ldots]$ with $i_{k,\ell}$ denoting the ℓ-th bit of the index I_k. Since the overcomplete expansion $\mathbf{G}$ adds redundancy to the data vector $\boldsymbol{U}$ the indices I_k show dependencies, which will in the following be modeled as a first-order stationary Gauss-Markov process with transition probabilities $P(I_k = \lambda \mid I_{k-1} = \mu)$ with $\mu, \lambda \in \mathscr{I}$.

The sequence $\boldsymbol{I}$ is transmitted over an AWGN channel with noise variance $\sigma_e^2 = \frac{N_0}{2E_s}$, where coherently detected binary-phase shift keying is assumed for the modulation. N_0 denotes the one-sided power spectral density and E_s is the transmit energy per codebit. Then the conditional p.d.f. of a received soft-bit $\hat{i}_{k,\ell}$ is Gaussian-distributed and can be written as

$$p(\hat{i}_{k,\ell} \mid i_{k,\ell}) = \frac{1}{\sqrt{2\pi}\sigma_e} \cdot \exp\left(-\frac{1}{2\sigma_e^2}(\hat{i}_{k,\ell} - \bar{i}_{k,\ell})^2\right) \quad (2)$$

with $\bar{i}_{k,\ell} = 1 - 2 \cdot i_{k,\ell}$.

3. DECODER STRUCTURE

At the decoder, first a soft-input soft-output source decoder (SISO) is applied to the received soft-bit vector $\hat{\boldsymbol{I}}$. The SISO decoder issues reliability information for the source hypotheses $I_k = \lambda$ at the output of the decoder in form of *a posteriori* probabilities (APPs). Here an index-based version of the classical BCJR algorithm [10] with the states $I_k = \lambda$ may be used as SISO decoder. However, for the sake of simplicity and in order to obtain a smaller system latency we restrict ourselves to just the forward recursion of the BCJR algorithm, where the APPs at the output of the SISO decoder $P(I_k = \lambda \mid \hat{\boldsymbol{I}}_0^k)$ are only conditioned on the received soft-bit vector $\hat{\boldsymbol{I}}_0^k = [\hat{i}_{1,1}, \hat{i}_{1,2}, \ldots, \hat{i}_{k,M}]$ up to the time instant k. By additionally considering the *a priori* index correlation $P(I_k = \lambda \mid I_{k-1} = \mu)$ due to the outer expansion for error resilience, the decoding rule can be written as

$$P(I_k = \lambda \mid \hat{\boldsymbol{I}}_0^k) = c_k\, p(\hat{I}_k \mid I_k = \lambda) \cdot \sum_{\mu=0}^{2^M-1} P(I_k = \lambda \mid I_{k-1} = \mu) P(I_{k-1} = \mu \mid \hat{\boldsymbol{I}}_0^{k-1}) \quad (3)$$

for $\lambda = 0, 1, \ldots, 2^M - 1$ with the constant c_k. Since the considered AWGN channel is memoryless the channel term $p(\hat{I}_k \mid I_k)$ can be obtained from (2) according to

$$p(\hat{I}_k \mid I_k) = \prod_{\ell=0}^{M-1} p(\hat{i}_{k,\ell} \mid i_{k,\ell}). \quad (4)$$

Using the obtained APPs from (3) we perform the following MAP decoding at the decoder output:

$$\lambda_{\text{map}} = \arg\max_{\lambda} P(I_k = \lambda \mid \hat{\boldsymbol{I}}_0^k).$$

Given λ_{map}, the corresponding element $\hat{Y}_k$ of the sequence $\hat{\boldsymbol{Y}}$ in Figure 1 is obtained according to $\hat{Y}_k = y_\lambda$, where y_λ denotes the quantizer reconstruction level corresponding to the index λ_{map}. Besides, the corresponding APP $P(I_k = \lambda_{\text{map}} \mid \hat{\boldsymbol{I}}_0^k)$ represents an element $P_{k,\text{map}}$ of the probability sequence $\boldsymbol{P}_{\text{map}}$.

For the further symbol reconstruction, we first construct a matrix $\mathbf{T}_{L\times N}$ which plays the role of a parity check matrix. For odd K, $\mathbf{T}_{L\times N}$ is the submatrix of $\mathbf{T}_N$ that corresponds to the L inserted zeros, such that

$$\mathbf{T}_{L\times N} \cdot \mathbf{y}_N = \mathbf{0}_L \quad (5)$$

where $\mathbf{y}_N$ is a length-N block of the original symbol sequence, and $\mathbf{0}_L$ is a vector of L zeros. For even K, the first $L-1$ rows of $\mathbf{T}_{L\times N}$ consist of the $L-1$ rows of $\mathbf{T}_N$ that correspond to the inserted zeros. The L-th row is the difference of the two rows of $\mathbf{T}_N$ that correspond to the repeated symbol, so that (5) is again satisfied.

Now we consider the vector

$$\mathbf{s} = \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N, \tag{6}$$

where $\hat{\mathbf{y}}_N$ denotes a block of N elements taken from $\hat{\boldsymbol{Y}}$. The vector $\mathbf{s}$ is called the *syndrome* and has a similar function as the syndrome in classical BCH codes over finite fields. However, in the method proposed in this paper we also have to take the effect of quantization into account. There are two points to mention:

1. In the presence of quantization, but without transmission errors, the vector $\mathbf{s}$ will, in general, be nonzero. Then the knowledge that the original symbols before quantization satisfy (5) helps to reduce the effects of quantization when reconstructing the final output $\hat{\boldsymbol{U}}$ [6]. The case of no transmission errors occurs when we already have perfect error correction by the SISO source decoder on the basis of the index correlation in $\boldsymbol{I}$.
2. In the presence of both quantization and transmission errors, the vector $\mathbf{s}$ will be significantly different from zero. We can then use the knowledge that $\mathbf{T}_{L\times N} \cdot \mathbf{y}_N = \mathbf{0}_L$ in order to simultaneously correct transmission errors and reduce the effects of quantization.

The decoding strategy is essentially analog to the decoding of binary block codes. We define the corrected symbols as

$$\tilde{\mathbf{y}} = \hat{\mathbf{y}}_N + \hat{\mathbf{e}} \tag{7}$$

and demand that

$$\mathbf{T}_{L\times N} \cdot \tilde{\mathbf{y}} = \mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}] = \mathbf{0}_L. \tag{8}$$

The question is then how to find the correction term $\hat{\mathbf{e}}$ that leads to the best estimate $\tilde{\mathbf{y}}$ for the true vector $\mathbf{y}_N$. Because of $N > L$ the linear system (8) is underdetermined and has infinitely many solutions for $\hat{\mathbf{e}}$.

For estimating $\hat{\mathbf{e}}$ subject to (8) from the information that is available at the output of the SISO decoder, we need the conditional p.d.f. $p(\hat{\mathbf{e}}|\sigma_e^2, \mathbf{P}_{\text{map}}, \hat{\mathbf{y}}_N)$. This density will, in general, be non-Gaussian. However, for the reason of simplicity and in order to obtain a linear estimator, we approximate this p.d.f by a Gaussian distribution. Moreover, we assume that the p.d.f. is independent of $\hat{\mathbf{y}}_N$ and that all components $\hat{e}_i$ of $\hat{\mathbf{e}}$ have zero mean and are mutually uncorrelated. The variances of the components $\hat{e}_i$, denoted by λ_i, are modeled to be dependent on the reliability information $P_{\text{map},i}$ and the normalized channel noise variance σ_e^2. Under the assumptions made above, the conditional p.d.f. of the correction term $\hat{\mathbf{e}}$ is given by

$$p(\hat{\mathbf{e}}|\sigma_e^2, P_{\text{map}}) = \big((2\pi)^N \prod_{i=1}^{N} \lambda_i\big)^{-\frac{1}{2}} \exp(-\frac{1}{2}\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}}) \tag{9}$$

where

$$\boldsymbol{\Lambda} = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_N], \tag{10}$$

$i = 1, 2, \ldots, N$. The quantities λ_i denote the variance of the true errors $e_i = \hat{y}_{N,i} - y_{N,i}$ and are approximated as $\tilde{\lambda}_i = \tilde{\lambda}_i(P_{\text{map},i}, \sigma_e^2)$ in the following.

The relationship between $\tilde{\lambda}_i$ and the terms $P_{\text{map},i}$ and σ_e^2 was found experimentally and approximated by a polynomial of the form

$$\lambda_i(P_{\text{map},i}, \sigma_e^2) = \sum_{k=0}^{\kappa} \alpha_k P_{\text{map},i}^k \tag{11}$$

where the coefficients α_k depend on σ_e^2. To determine the coefficients α_k, trials were made where the mean squared error $E\{|\hat{y}_i - y_i|^2\}$ was collected for $P_{\text{map},i}$ lying in intervals $[(q-1)/Q,\ q/Q]$, $q = 1, 2, \ldots, Q$ with $Q = 20$ and for various fixed values of σ_e^2. In a second step, a least-squares polynomial fit of the collected data was carried out to determine the coefficient sets α_k, $k = 0, 1, \ldots, \kappa$ for each value of σ_e^2, where an order $\kappa = 8$ for the approximation polynomial was chosen.

Maximizing (9) through the choice of $\hat{\mathbf{e}}$ is equivalent to minimizing $\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}}$, so that we obtain the following optimization problem:

$$\begin{array}{ll} \text{minimize} & \hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}} \\ \text{subject to} & \mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}] = \mathbf{0}_L. \end{array} \tag{12}$$

The solution can be found via the Lagrange multiplier method. We then have to minimize

$$\hat{\mathbf{e}}^T \boldsymbol{\Lambda}^{-1} \hat{\mathbf{e}} + \mu^T (\mathbf{T}_{L\times N} \cdot [\hat{\mathbf{y}}_N + \hat{\mathbf{e}}]) \tag{13}$$

through the choice of $\hat{\mathbf{e}}$ and vector μ. The solution to this problem is given by

$$\begin{bmatrix} \hat{\mathbf{e}} \\ \mu \end{bmatrix} = -\begin{bmatrix} 2\boldsymbol{\Lambda}^{-1} & \mathbf{T}_{L\times N}^T \\ \mathbf{T}_{L\times N} & \mathbf{0}_{L\times L} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N \end{bmatrix}, \tag{14}$$

and after partitioned inversion, we obtain

$$\hat{\mathbf{e}} = -\boldsymbol{\Lambda} \mathbf{T}_{L\times N}^T [\mathbf{T}_{L\times N} \boldsymbol{\Lambda} \mathbf{T}_{L\times N}^T]^{-1} \mathbf{T}_{L\times N} \cdot \hat{\mathbf{y}}_N. \tag{15}$$

Given $\hat{\mathbf{e}}$, we determine $\tilde{\mathbf{y}}$ from (7). A reconstructed block $\hat{\mathbf{u}}$ of the sequence $\hat{\boldsymbol{U}}$ at the output of the reconstruction stage is finally obtained as

$$\hat{\mathbf{u}} = \mathbf{G}^{\dagger} \tilde{\mathbf{y}} \tag{16}$$

where $\mathbf{G}^{\dagger}$ denotes the pseudo-inverse of $\mathbf{G}$.

It is worth to mention that the correction term $\hat{\mathbf{e}}$ only has an influence on the final output $\hat{\mathbf{u}}$ when $\boldsymbol{\Lambda}$ is not a multiple of the identity matrix, because due to the orthogonality of $\mathbf{T}_N$ and the given construction of $\mathbf{P}$, we have $\mathbf{G}^{\dagger} \mathbf{T}_{L\times N}^T = \mathbf{0}_{K\times L}$.

4. SIMULATION RESULTS

In order to verify the performance of the resulting transmission system simulations were carried out for an AR(1) input process $\boldsymbol{U}$ with correlation coefficient a and a block length of 160000 source symbols. The frame expansion uses the parameters $K = 16$ and $N = 32$, and the subsequent scalar uniform quantization has a resolution of $M = 5$ bit. We employ different unitary transforms $\mathbf{T}$, namely, the DFT, the DCT-II, and the DHT. The results are displayed in Figures 2 and 3 where the reconstruction SNR at the decoder output is plotted over the channel parameters E_b/N_0 with $E_b = E_s/R$. The overall code rate R is given as $R = K/N$.

We can see from Figure 2, which shows the result for an uncorrelated input process with $a = 0$, that especially for the

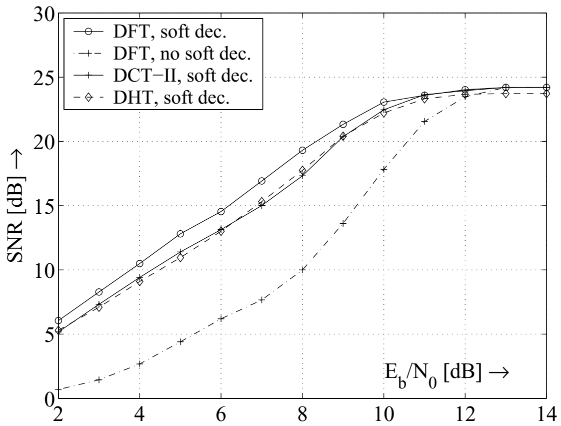

Figure 2: Reconstruction SNR for an i.i.d. input process (Parameters: $K = 16$, $N = 32$, $M = 5$ bit, block length 80000 source symbols)

DFT a strong gain is obtained when SISO source decoding is additionally used prior to the decoding of the overcomplete expansion. This is due to the fact that only for the DFT a reasonable amount of index correlation is present after the overcomplete expansion if the input process $\boldsymbol{U}$ is uncorrelated, which can be verified from the entropies listed in Table 1.

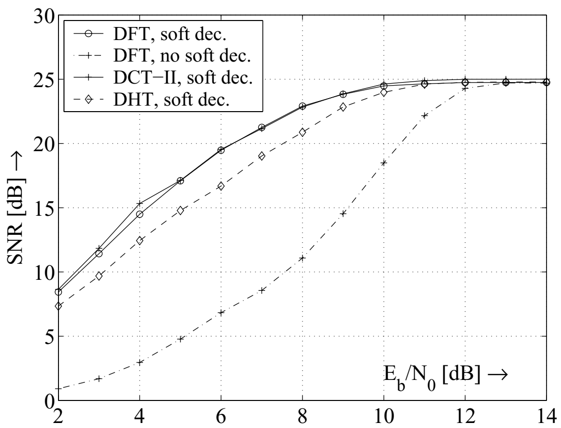

Figure 3: Reconstruction SNR for a strongly correlated AR(1) input process with $a = 0.9$ (Parameters: $K = 16$, $N = 32$, $M = 5$ bit, block length 80000 source symbols)

Figure 3 depicts the results for a strongly correlated AR(1) process with $a = 0.9$. Compared to Figure 2, here the SNR gain by using additional SISO source decoding is higher due to the source symbol correlation already inherent in the input sequence $\boldsymbol{U}$. Here, the DFT and also the DCT-II outperform the DHT.

5. CONCLUSIONS

In this paper we have extended the syndrome-based decoding of a real-valued block channel code based on overcomplete expansions by an additional soft-input soft-output (SISO) source decoding stage. This SISO decoder is capable of exploiting the source index correlation introduced by the overcomplete expansion for additional error protection. Furthermore, we have shown that the reliability information for the estimated source symbols at the output of the SISO decoder

Transform	a	$H(I_k)$	$H(I_k\,\vert\,I_{k-1})$
DFT	0	3.680	3.341
	0.9	3.680	2.207
DCT-II	0	3.764	3.762
	0.9	3.908	2.560
DHT	0	3.753	3.752
	0.9	3.817	2.857

Table 1: Measured entropies $H(I_k)$ and $H(I_k\,|\,I_{k-1})$ after frame expansion with $K = 16$, $N = 32$ and quantization with $M = 5$ bits for an AR(1) process with correlation coefficient a and different transforms

can be used for improving the result of the subsequent syndrome decoding. When using the DFT as the underlying transform for the overcomplete expansion a significant SNR gain is obtained for the reconstructed sequence at the decoder output even if the source symbols are uncorrelated.

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