IMPLEMENTATION OF BIORTHOGONAL COSINE-MODULATED FILTER BANKS WITH FIXED-POINT ARITHMETIC

Tanja Karp

Texas Tech University
ECE Dept.
Lubbock, TX 79409, USA
tanja.karp@ttu.edu

Alfred Mertins

University of Wollongong
School of Elec., Comp., and Tele. Eng.
Wollongong, NSW 2522, Australia
mertins@uow.edu.au

ABSTRACT

In this paper, we consider the implementation of the prototype filter of biorthogonal cosine-modulated filter banks on processors with fixed-point arithmetic. The realization is based on zero-delay and maximum-delay matrices. We show that the perfect reconstruction property of the filter bank is not affected by quantization, rounding and overflow. We also demonstrate how the latter operations influence the frequency selectivity of the filter bank.

1. INTRODUCTION

Cosine-modulated filter banks are popular because of their low computational cost and their perfect reconstruction (PR) property. However, when implementing such a filter bank on a processor with finite-precision arithmetic, the prototype and the modulating sequences usually have to be quantized and the PR property gets lost. Rounding the results of each multiplication and addition to the available wordlength results in further reconstruction errors. It is therefore of significant interest to have filter banks that allow PR also with finite-precision arithmetic.

Biorthogonal cosine-modulated filter banks have been studied in [1, 2, 3]. Other than paraunitary filter banks they allow to design the system delay independently of the filter length, thus, resulting in a better stopband attenuation and a smaller transition bandwidth for a given system delay than paraunitary filter banks. In audio coding applications, it has been shown that biorthogonal filter banks can significantly reduce pre-echoes [2, 3].

In this paper, we concentrate on the implementation of the prototype filter of biorthogonal cosine-modulated filter banks on a fixed-point digital signal processor (DSP). For the realization of the modulation matrix, different algorithms have been published in [4, 5, 6]. The design of integer-coefficient prototype filters for paraunitary filter banks has been treated in [7, 8]. In this paper, however, we consider prototypes with real-valued coefficients and include coefficient quantization as part of the implementation structure.

The paper is organized as follows: First, we recall basic properties of cosine-modulated filter banks and show how they can be realized using zero-delay and maximum-delay matrices. We then show that coefficient quantization and rounding operations after each multiplication and addition does not affect the PR property. Finally, we demonstrate how these nonlinear operations affect the amplitude spectrum of the subband signals.

2. COSINE MODULATED-FILTER BANKS

We consider critical subsampling, an even number of bands (M), use of the same FIR prototype $p(n)$ for both, analysis and synthesis, and an overall delay of $D = 2sM + 2M - 1$ with s being an integer. Note that these are the most common choices in the design of cosine-modulated filter banks. The analysis and synthesis filters, denoted as $h_k(n)$ and $f_k(n)$, $k = 0, \dots, M-1$, $n = 1, \dots, N-1$, are derived as

$$h_k(n) = 2p(n)\,\cos(\frac{\pi}{M}(k+0.5)(n-D/2)+\theta_k) \quad (1)$$

$$f_k(n) = 2p(n)\,\cos(\frac{\pi}{M}(k+0.5)(n-D/2)-\theta_k) \quad (2)$$

with $\theta_k = (-1)^k\frac{\pi}{4}$. The traditional polyphase realization of a cosine modulated filter bank [9] is given in Figure 1. $G_\ell(z)$ and $K_\ell(z)$, $\ell = 0, \dots 2M-1$, denote the type-1 and type-2 polyphase components [9] of the prototype filter $P(z)$, respectively, and the modulation matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ write for $0 \le k < M$ and $0 \le \ell < 2M$:

$$[\mathbf{C}_1]_{k,\ell} = 2\cos((k+0.5)\frac{\pi}{M}(\ell - \frac{D}{2}) + \theta_k) \quad (3)$$

$$[\mathbf{C}_2]_{k,\ell} = 2\cos((k+0.5)\frac{\pi}{M}(2M-1-\ell-\frac{D}{2}) - \theta_k) \quad (4)$$

In [10] we have shown that the computational cost of the polyphase realization can be approximately halved when

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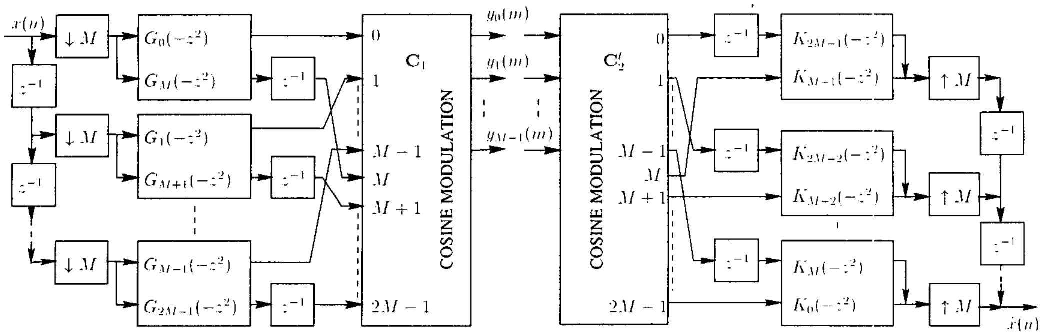

Figure 1: Traditional polyphase realization

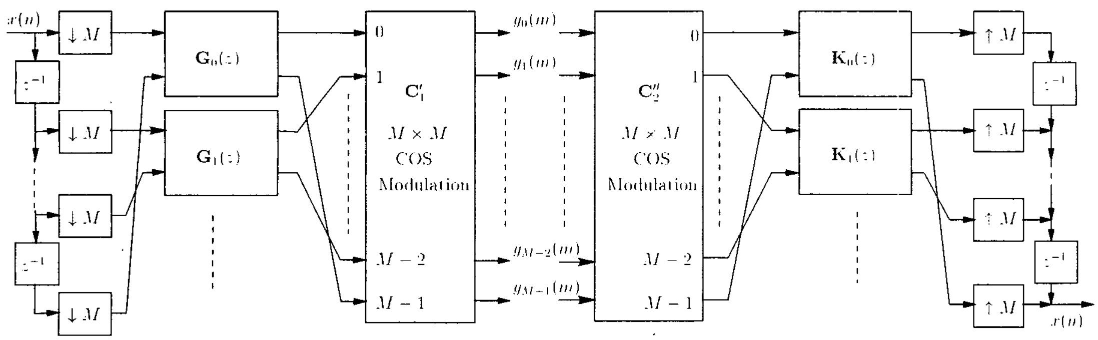

Figure 2: Efficient polyphase realization

taking into consideration symmetry properties of the modulation matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ which results in the implementation shown in Figure 2 with

$$\mathbf{G}_\ell(z) = \begin{bmatrix} G_\ell(-z^2) & (-1)^s G_{M-1-\ell}(-z^2) \\ (-1)^{s-1} z^{-1} G_{\ell+M}(-z^2) & z^{-1} G_{2M-1-\ell}(-z^2) \end{bmatrix} \tag{5}$$

$$\mathbf{K}_\ell(z) = \begin{bmatrix} z^{-1} K_{2M-1-\ell}(-z^2) & (-1)^{s-1} K_{M-1-\ell}(-z^2) \\ (-1)^s z^{-1} K_{\ell+M}(-z^2) & K_\ell(-z^2) \end{bmatrix} \tag{6}$$

for $\ell = 0, \dots M/2 - 1$ and

$$[\mathbf{C}_i']_{k,\ell} = \begin{cases} [\mathbf{C}_i]_{k,\ell} & \ell = 0, \dots, M/2 - 1 \\ [\mathbf{C}_i]_{k,\ell+M} & \ell = M/2, \dots, M-1 \end{cases}, \quad i = 1, 2 \tag{7}$$

3. ZERO-DELAY AND MAXIMUM-DELAY MATRICES

If the filter bank is PR,

$$\mathbf{K}_\ell(z)\mathbf{G}_\ell(z) = \frac{(-1)^s z^{-2s-1}}{2M} \mathbf{I}_2, \quad 0 \le \ell < M/2 \tag{8}$$

holds true, and for contiguous prototype filters, $\mathbf{G}_\ell(z)$ and $\mathbf{K}_\ell(z)$ can always be realized in the following way [10, 3]:

$$\mathbf{G}_\ell(z) = \prod_{j=1}^{j_0} \mathbf{D}_{\ell,j}(z) \prod_{i=1}^{i_0} \mathbf{B}_{\ell,i}(z) \cdot \mathbf{G}_{\ell,ini}(z) \tag{9}$$

$$\mathbf{K}_\ell(z) = \mathbf{K}_{\ell,ini}(z) \prod_{i=i_0}^{1} \mathbf{B}_{\ell,i}^{-1}(z) \prod_{j=j_0}^{1} (z^{-2}\mathbf{D}_{\ell,j}^{-1}(z)), \tag{10}$$

where $j_0 = 2s$ for a fixed value of s in (8) and and i_0 such that the desired filter length N is met. The matrices $\mathbf{B}_{\ell,i}(z)$, $\mathbf{D}_{\ell,j}(z)$, and $\mathbf{G}_{\ell,ini}(z)$ are called zero-delay, maximum delay, and initialization matrices, respectively, and have the following form:

$$\mathbf{B}_{\ell,i}(z) = \begin{bmatrix} 0 & 1 \\ 1 & b_{\ell,i} z^{-1} \end{bmatrix}, \quad \mathbf{B}_{\ell,i}^{-1}(z) = \begin{bmatrix} -b_{\ell,i} z^{-1} & 1 \\ 1 & 0 \end{bmatrix} \tag{11}$$

$$\mathbf{D}_{\ell,j}(z) = \begin{bmatrix} d_{\ell,j} & z^{-1} \\ z^{-1} & 0 \end{bmatrix}, \quad z^{-2}\mathbf{D}_{\ell,j}^{-1}(z) = \begin{bmatrix} 0 & z^{-1} \\ z^{-1} & -d_{\ell,j} \end{bmatrix} \tag{12}$$

$$\mathbf{G}_{\ell,ini}(z) = \frac{(-1)^s}{2M}\begin{bmatrix} 1 & 0 \\ \tilde{g}_{\ell,0}\, z^{-1} & z^{-1} \end{bmatrix}\begin{bmatrix} 1 & \tilde{g}_{\ell,1} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \tilde{g}_{\ell,2} & 1 \end{bmatrix} \tag{13}$$

$$\mathbf{K}_{\ell,ini}(z) = \frac{1}{2M}\begin{bmatrix} 1 & 0 \\ -\tilde{g}_{\ell,2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tilde{g}_{\ell,1} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} z^{-1} & 0 \\ -\tilde{g}_{\ell,0}\, z^{-1} & 1 \end{bmatrix} \tag{14}$$

4. QUANTIZATION

Each of the matrices in (11)-(14) contains only one coefficient which is subject to quantization. Since the inverse matrix contains the same coefficient, coefficient quantization does not alter the PR property of the filter bank as long as the same quantization scheme is applied. However, coefficient quantization does affect the frequency response of the prototype filter, as we will see in Section 5.

When multiplying two fixed-point numbers the wordlength of the product equals the sum of both wordlengths and rounding becomes necessary to reduce the wordlength to the desired one (usually the wordlength of the input data). Figure 3 shows the realization of a zero delay matrix with Matlab Simulink Fixed-Point Blockset. Figure 4 shows the structure for an inverse zero-delay matrix.

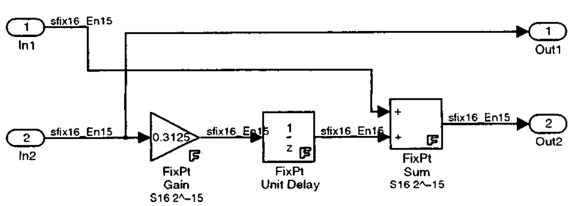

Figure 3: Zero-delay matrix using Matlab Simulink Fixed-Point Blockset. Wordlength: 16 bit for both signal and coefficient.

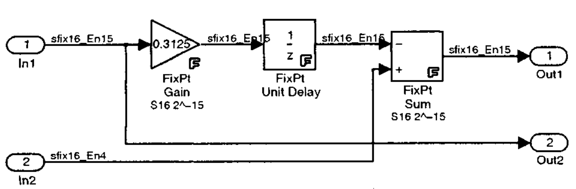

Figure 4: Inverse zero-delay matrix using Matlab Simulink Fixed-Point Blockset. Wordlength: 16 bit for both signal and coefficient.

In Figures 3 and 4, we used signed fixed point representation with a wordlength of 16 bit for both the signal as well as the coefficients. As long as we apply the same rounding scheme (i.e. towards ceiling, floor, nearest) in the zero-delay matrix and its inverse, the PR property of the filter bank is not affected by the rounding operation, since we first add the rounded signal in the analysis and then subtract exactly the same signal in the synthesis. For the same reason, a possible overflow in the adder is harmless to the PR property as long as the adder does not saturate in this case. The same holds true for maxmimum-delay as well as the intialization matrix since they have a similar structure. Note that the same argumentation has been followed in [11] for the design of integer-to-integer wavelet transforms.

5. DESIGN EXAMPLES

Although nonlinear operations such as quantization, rounding and overflow do not affect the PR property of the filter bank they can severely alter its frequency selectivity. In the following we give design examples using a white i.i.d. input signal. An estimate for the amplitude spectrum of the subbands prior to subsampling is obtained by averaging the DFT of non-overlapping rectangular signal windows of length 1024. We only consider the influence of the polyphase realization of the prototype and assume that the modulation matrix is perfect. The prototype implementation is simulated using the Fixed Point Blockset from Matlab Simulink.

5.1. Example 1

For a filter bank with $M = 8$ subbands, we implement a lowdelay prototype filter of length $N = 32$ that causes an overall system delay of $D = 15$. For this setting, $\mathbf{G}_\ell(z)$ according to (9) consists of the initialization matrix and two zero-delay matrices. The coefficients of the original prototype filter are given in Table 1.

Table 1: Coefficients of the floating-point prototype filter

$\tilde{g}_{01}$	−0.74788545432548	$\tilde{g}_{21}$	−0.87530466500673
$\tilde{g}_{02}$	0.91446400442459	$\tilde{g}_{22}$	0.69704618303368
$\tilde{g}_{03}$	−0.91180806866778	$\tilde{g}_{23}$	−0.93566919243152
b_{01}	0.33375425498257	b_{21}	0.66057082567363
b_{02}	0.47272551599654	b_{22}	0.18343353417840
$\tilde{g}_{11}$	−0.77512756844701	$\tilde{g}_{31}$	−1.16561172679350
$\tilde{g}_{12}$	0.81046800456288	$\tilde{g}_{32}$	0.57772283002570
$\tilde{g}_{13}$	−0.92816707422974	$\tilde{g}_{33}$	−0.93364838915293
b_{11}	0.45863225672890	b_{31}	1.05192947158651
b_{12}	0.31703578350340	b_{32}	0.07966367878380

Figure 5 shows the normalized amplitude spectrum of the lowpass band prior to subsampling. The white i.i.d. input signal is in the range $x \in [-1, 1]$ and is linearly quantized to a wordlength of 16 bit. The filter coefficients are also quantized to 4, 8, and 16 bit wordlength. We can see from Figure 5 that the filter coefficients are not very sensitive to quantization. The result for 8, 16 bit and the ideal case (no quantization, ideal white noise) superpose. Only when quantizing the coefficients to 4 bit wordlength an increase of the power in the stopband can be observed.

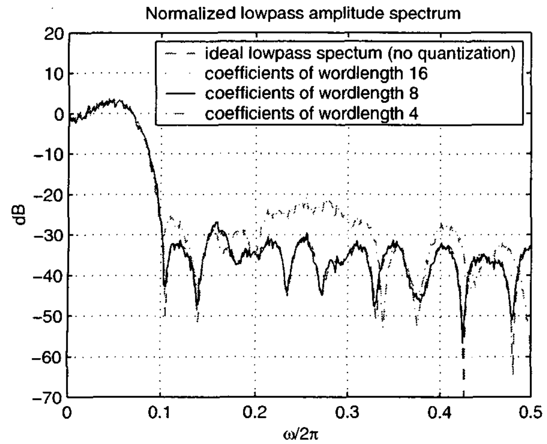

Figure 5: Influence of coefficient quantization on the lowpass amplitude spectrum

5.2. Example 2

The variance of the input signal in Example 1 was chosen such that hardly any overflow occurred in the analysis filter bank. The influence of overflow can be observed in Figure 6 where the normalized lowpass spectrum is given for different ranges of the input signal without changing the scaling of the quantization. For $x \in [-2, 2]$ occational overflow could be observed during simulation. This does not alter the frequency behavior compared to the ideal case. However, when severe overflow occurs as in the case for $x \in [-3, 3]$ the power in the stopband increases dramatically.

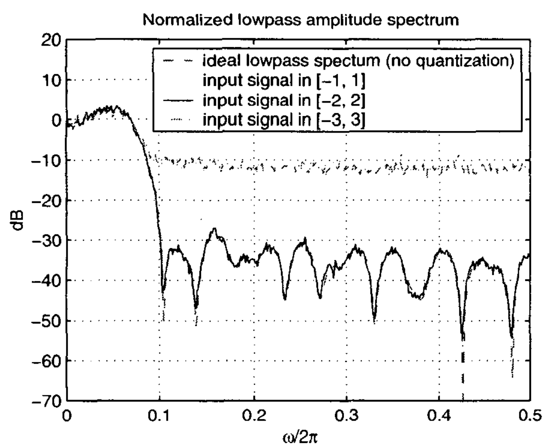

Figure 6: Influence of overflow on the lowpass spectrum

6. CONCLUSIONS

We have given a structure for the implementation of biorthogonal cosine-modulated filter banks with fixed-point arithmetic. Our approach is based on zero-delay and maximum delay matrices and keeps the perfect reconstruction property despite quantization, rounding and overflow. Design examples demonstrate that the influence of overflow on the frequency selectivity is much greater than that of quantization and rounding.

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