

BIORTHOGONAL COSINE-MODULATED FILTER BANKS WITHOUT DC LEAKAGE

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ABSTRACT

In this paper, we present a structure for implementing the polyphase filters of biorthogonal modulated filter banks that automatically guarantees perfect reconstruction of the filter bank and furthermore allows to specify the values of the filters' frequency responses at certain frequencies. Thus, modulated filter banks without DC leakage can be designed. The new structure is based on lifting schemes for the polyphase filters and DC leakage can be avoided very easily when reducing the number of lifting coefficients that can be freely chosen and used for filter optimization. The great advantage of the new method is that we do not have to take constraints into consideration when optimizing the prototype filter, but PR and specified zeros are structure inherent.

1. INTRODUCTION

Cosine-modulated filter banks have been studied extensively in literature within the last 10 years. They have shown to provide a very efficient implementation based on a prototype filter and a fast cosine transform. The filter bank can be designed such as to be paraunitary or to result in a low system delay. In the latter case, also called biorthogonal case, the overall system delay can be chosen independently (within some fundamental limits) of the filter length and the number of subbands. As has been shown in [1, 2] one can still use a common prototype for the analysis and synthesis. Design methods for the prototype mainly base on two different philosophies: It is either possible to use a constrained optimization (with the PR conditions as constraints) of the prototype's frequency response. This has led to the QCLS-algorithm [3, 4, 1]. A second possibility consists of deriving a structure that automatically guarantees PR of the filter bank. In the paraunitary case, such a structure is given by the well known lattice-structure [5] containing rotations for the lattice coefficients. For biorthogonal filter banks such structures have been derived in [6, 2, 7] and non-linear, non-constrained optimization methods are used for the prototype design. Compared to the QCLS-algorithm, the latter algorithms offer the advantage to be robust against coefficient quantization, thus allowing an efficient filter implementation with integer-valued coefficients [2, 8, 9].

We here extend the method proposed in [7] based on the use of lifting schemes [10, 11] for the polyphase filters' implementation in order to design filters with specified zeros at certain frequencies. This allows the design of cosine-modulated filter banks without DC leakage, meaning that the DC component of an input signal

only affects the subband signal in the lowest band. In such a filter bank all analysis filters apart from the lowpass filter must have a zero at frequency zero. This is an additional necessary condition to those for PR and we show that it can be incorporated in the lifting schemes, resulting in a structurally inherent feature.

In the following we consider cosine-modulated filter banks with M subbands and restrict ourselves to the case where the filter length N is a multiple of $2M$, i.e. $N = 2mM$ and the overall system delay D given by $D = 2sM + M - 1$. As shown in [4] the analysis and synthesis filters, $H_k(z)$ and $F_k(z)$, of such a filter bank can be derived from a prototype $P(z)$ by DCT-IV modulation:

$$h_k(n) = 2p(n) \cos \left[(2k+1) \frac{\pi}{2M} \left(n - \frac{D}{2} \right) + (-1)^k \frac{\pi}{4} \right], \quad (1)$$

$$f_k(n) = 2p(n) \cos \left[(2k+1) \frac{\pi}{2M} \left(n - \frac{D}{2} \right) - (-1)^k \frac{\pi}{4} \right], \quad (2)$$

and the PR constraints can be expressed on the prototype's type-1 polyphase filters $g_k(m) = p(2mM + k)$, $k = 0, \dots, 2M - 1$, according to

$$\begin{bmatrix} G_\ell(z) & G_{M+\ell}(z) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} G_{2M-1-\ell}(z) \\ G_{M-1-\ell}(z) \end{bmatrix} = \frac{z^{-s}}{2M} \quad (3)$$

for $0 \leq \ell < M$ and $D = 2sM + M - 1$.

2. COSINE-MODULATED FILTER BANKS WITHOUT DC LEAKAGE

Figure 1 shows the magnitude frequency responses of the analysis filters for an M -channel cosine-modulated filter bank as well as the prototype.

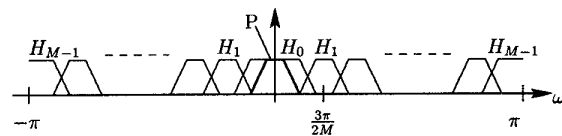


Figure 1: Magnitude frequency responses of analysis filters and prototype

The cosine-modulated filter bank is free of DC leakage if the analysis filters $H_k(\omega)$, $k > 0$, have at least one zero at frequency zero, i.e. $H_k(0) = 0$ for $k = 1, \dots, M - 1$. The lowpass filter has to satisfy $H_0(0) = 1$.

Applying the discrete Fourier transform

$$H(\omega) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}, \quad (4)$$

and using (1) we obtain for $H_k(0) = \sum_{n=0}^{N-1} h_k(n)$ the following relationship:

$$H_k(0) = \sum_{n=0}^{N-1} 2p(n)c(k, n) \quad (5)$$

with

$$c(k, n) = \cos \left[(2k+1) \frac{\pi}{2M} (n - sM - \frac{M-1}{2}) + (-1)^k \frac{\pi}{4} \right]$$

Taking into consideration the $4M$ periodicity in n of $c(k, n)$, as well as $c(k, n) = -c(k, n + 2M)$ and changing the summation index n into $n = 2\nu M + \mu$, we obtain

$$H_k(0) = \sum_{\mu=0}^{2M-1} \sum_{\nu=0}^{m-1} \underbrace{2p(2\nu M + \mu)}_{g_\mu(\nu)} (-1)^\nu c(k, \mu)$$

which may also be written as

$$H_k(0) = \sum_{\mu=0}^{2M-1} 2c(k, \mu) \alpha_\mu \quad \text{with} \quad \alpha_\mu = \sum_{\nu=0}^{m-1} g_\mu(\nu) (-1)^\nu, \quad (6)$$

$k = 0, \dots, M-1$, or in matrix form as

$$\mathbf{C}\boldsymbol{\alpha} = \mathbf{h} \quad (7)$$

with

$$\begin{aligned} [\mathbf{C}]_{k,\mu} &= c(k, \mu), \quad k = 0, \dots, M-1, \mu = 0, \dots, 2M-1 \\ \boldsymbol{\alpha} &= [\alpha_0, \dots, \alpha_{2M-1}]^T \\ \mathbf{h} &= [H_0(0), \dots, H_{M-1}(0)]^T \end{aligned}$$

In order to prevent DC leakage we have to choose $\mathbf{h} = [1, 0, \dots, 0]^T$.

In the following section we present an implementation of the polyphase filters $g_\ell(n)$ that automatically guarantees PR of the filter bank and is especially suited to incorporate the condition (7) in order to obtain the desired zeros. Since (7) is an underdetermined linear set of equations, it has an infinite number of solutions. However, we restrict us to those solutions satisfying

$$\alpha_\mu \alpha_{2M-1-\mu} + \alpha_{\mu+M} \alpha_{M-1-\mu} = \frac{(-1)^\mu}{2M}, \quad (8)$$

because they can be easily incorporated in the filter design method based on lifting schemes. One possible solution for α_μ writes:

$$\alpha_\mu = \frac{1}{\sqrt{2M}} \cos \left(\frac{\pi}{2M} \left(\mu - \frac{D}{2} \right) \right), \quad \mu = 0, \dots, 2M-1. \quad (9)$$

3. THE LIFTING SCHEME

The lifting scheme was introduced in [10, 11] in order to construct biorthogonal and second generation wavelets. It has been proved in [7] that the polyphase filters of PR modulated filter banks can also be designed using lifting and that this structure automatically guarantees the PR property of the bank.

The main idea of lifting is demonstrated in Figure 2. One starts with a set of short filters $G_*(z)$ satisfying the PR constraint (3) and increases the length of two filters by means of a new filter

$A(z)$. Since the same signal that is added to the original subbands by this step on the analysis side is subtracted on the synthesis side, we keep the PR property for any $A(z)$. In terms of the PR conditions this means that the identity matrix in (3) is replaced by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ A(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -A(z) & 1 \end{bmatrix}$$

A lifting step is typically followed by a dual lifting step where the length of the remaining two filters is increased. Overall, one alternates lifting and dual lifting to construct filters of the desired length.

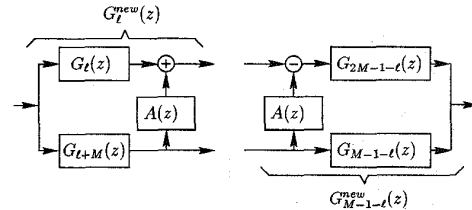


Figure 2: Construction of new polyphase filters using lifting

While the structure in Figure 2 increases the filter length and keeps the delay fixed, other schemes may be considered where both, the filter length and the delay, are increased.

3.1. Starting point

As a starting point for the construction of a cosine-modulated filter bank without DC leakage we take the length-1 polyphase filters as $G_\ell(z) = \tilde{g}_\ell$, $\ell = 0, \dots, 2M-1$, that satisfy the PR constraint (3) when being connected with some delay:

$$\begin{bmatrix} \tilde{g}_\ell & \tilde{g}_{M+\ell} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{g}_{2M-1-\ell} \\ \tilde{g}_{M-1-\ell} \end{bmatrix} z^{-s} = \frac{z^{-s}}{2M} \quad (10)$$

In the following we will show that the values g_ℓ cannot be chosen arbitrarily but will be determined by condition (9) imposed on the final length- m polyphase filters to prevent DC leakage. Note that for length-1 filters we have $\alpha_\ell = \tilde{g}_\ell$.

3.2. Increasing the Filter Length

We can now increase the polyphase filter length using the lifting scheme. With one lifting or dual lifting step, we always increase the polyphase filters by one tap, thus the prototype by $2M$ taps.

3.2.1. Zero-Delay Lifting

Let us start with a set of length-1 polyphase filters $G_\ell(z)$ that satisfy (10). In order to obtain new polyphase filters $G_\ell^{new}(z)$ and $G_{M-1-\ell}^{new}(z)$, we replace the identity matrix in (10) by a product of a matrix \mathbf{A} and its inverse:

$$\mathbf{A}\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 \\ az^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -az^{-1} & 1 \end{bmatrix} \quad (11)$$

This results in polyphase filters being one tap longer:

$$G_\ell^{new}(z) = G_\ell(z) + az^{-1}G_{M+\ell}(z), \quad (12)$$

$$G_{M-1-\ell}^{new}(z) = -az^{-1}G_{2M-1-\ell}(z) + G_{M-1-\ell}(z). \quad (13)$$

In a similar way, for the remaining two polyphase filters in (10) we can increase the length by one when applying a so-called dual lifting step after the lifting step and expressing the identity matrix in (10) by

$$\mathbf{B}\mathbf{B}^{-1} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix} \quad (14)$$

yielding

$$G_{M+\ell}^{new}(z) = b G_{\ell}^{new}(z) + G_{M+\ell}(z), \quad (15)$$

$$G_{2M-1-\ell}^{new}(z) = G_{2M-1-\ell}(z) - b G_{M-1-\ell}^{new}(z). \quad (16)$$

After these two steps all polyphase filters are increased by one tap. The values a and b are free parameters that will be used for optimization of the prototypes frequency response. From (12) we obtain for $\alpha_{\ell}^{new} = \sum_n g_{\ell}^{new}(n)(-1)^n$:

$$\begin{aligned} \alpha_{\ell}^{new} &= \sum_n (g_{\ell}(n)(-1)^n + a g_{M+\ell}(n-1)(-1)^n) \\ &= \sum_n g_{\ell}(n)(-1)^n - a \sum_n g_{M+\ell}(n)(-1)^n \\ &= \alpha_{\ell} - a \alpha_{M+\ell} \end{aligned} \quad (17)$$

and similarly:

$$\alpha_{M-1-\ell}^{new} = \alpha_{M-1-\ell} + a \alpha_{2M-1-\ell} \quad (18)$$

$$\alpha_{M+\ell}^{new} = \alpha_{M+\ell} + b \alpha_{\ell}^{new} \quad (19)$$

$$\alpha_{2M-1-\ell}^{new} = \alpha_{2M-1-\ell} - b \alpha_{M-1-\ell}^{new} \quad (20)$$

In matrix notation, we have

$$[\alpha_{\ell}^{new} \quad \alpha_{M+\ell}^{new}] = [\alpha_{\ell} \quad \alpha_{M+\ell}] \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} \alpha_{2M-1-\ell}^{new} \\ \alpha_{M-1-\ell}^{new} \end{bmatrix} = \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} \alpha_{2M-1-\ell} \\ \alpha_{M-1-\ell} \end{bmatrix} \quad (22)$$

This kind of lifting is called zero-delay lifting since we do increase the filter length but keep the overall system delay constant when performing a lifting and a dual lifting step. The upper procedure can be iterated.

3.2.2. Maximum-Delay Lifting

A second way to obtain new polyphase filters is to incorporate a maximal part of the necessary delay on the right-hand side of (3) into the lifting scheme. We start with a set of polyphase filters $G_{\star}(z)$ satisfying:

$$[G_{\ell}(z) \quad G_{M+\ell}(z)] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} G_{2M-1-\ell}(z) \\ G_{M-1-\ell}(z) \end{bmatrix} z^{-a} = \frac{z^{-s}}{2M} \quad (25)$$

with $a \leq s$, and replace $z^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ in the upper equation by

$$z^{-1} \mathbf{C} \mathbf{C}^{-1} = \begin{bmatrix} z^{-1} & 0 \\ c & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c & z^{-1} \end{bmatrix} \quad (26)$$

for lifting, and by

$$z^{-1} \mathbf{D} \mathbf{D}^{-1} = \begin{bmatrix} 1 & d \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} z^{-1} & -d \\ 0 & 1 \end{bmatrix} \quad (27)$$

for the dual lifting step, yielding

$$G_{M+\ell}^{new}(z) = z^{-1} G_{\ell}(z) + c G_{M+\ell}(z) \quad (28)$$

$$G_{2M-1-\ell}^{new}(z) = -c G_{2M-1-\ell}(z) + z^{-1} G_{M-1-\ell}(z) \quad (29)$$

and

$$G_{M+\ell}^{new}(z) = d G_{\ell}^{new}(z) + z^{-1} G_{M+\ell}(z), \quad (30)$$

$$G_{2M-1-\ell}^{new}(z) = z^{-1} G_{2M-1-\ell}(z) - d G_{M-1-\ell}^{new}(z). \quad (31)$$

For the α_{\star}^{new} we obtain in matrix notation:

$$[\alpha_{\ell}^{new} \quad \alpha_{M+\ell}^{new}] = [\alpha_{\ell} \quad \alpha_{M+\ell}] \begin{bmatrix} -1 & 0 \\ c & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & -1 \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} \alpha_{2M-1-\ell}^{new} \\ \alpha_{M-1-\ell}^{new} \end{bmatrix} = \begin{bmatrix} -1 & -d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c & -1 \end{bmatrix} \begin{bmatrix} \alpha_{2M-1-\ell} \\ \alpha_{M-1-\ell} \end{bmatrix} \quad (33)$$

The PR constraints (25) for the new polyphase filters write:

$$[G_{\ell}^{new}(z) \quad G_{M+\ell}^{new}(z)] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} G_{2M-1-\ell}^{new}(z) \\ G_{M-1-\ell}^{new}(z) \end{bmatrix} z^{-a+2} = \frac{z^{-s}}{2M}$$

The polyphase filters' length is increased by one tap and the delay parameter a reduced by two. Maximum-delay lifting can be iterated while $a^{new} = a - 2$ is at least two.

3.2.3. Single-Delay Lifting

If the delay parameter s is odd, maximum-delay lifting ends with $a^{new} = 1$. The remaining single delay step z^{-1} on the left-hand side of (25) can still be incorporated in the lifting scheme when combining a zero-delay lifting step with a maximum-delay dual lifting step, yielding new polyphase filters $G_{\ell}^{new}(z)$ and $G_{M-1-\ell}^{new}(z)$ according to (12) and (13), and polyphase filters $G_{M+\ell}^{new}(z)$ and $G_{2M-1-\ell}^{new}(z)$ according to (30) and (31). The values α_{\star}^{new} write:

$$[\alpha_{\ell}^{new} \quad \alpha_{M+\ell}^{new}] = [\alpha_{\ell} \quad \alpha_{M+\ell}] \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & -1 \end{bmatrix} \quad (34)$$

$$\begin{bmatrix} \alpha_{2M-1-\ell}^{new} \\ \alpha_{M-1-\ell}^{new} \end{bmatrix} = \begin{bmatrix} -1 & -d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} \alpha_{2M-1-\ell} \\ \alpha_{M-1-\ell} \end{bmatrix} \quad (35)$$

3.3. Final Implementation

The final implementation of the polyphase filters is described in (23). l_{\max} is set to 0 for even s and to 1 for odd s . The free variables for the non-linear non-constrained optimization are given by the a_i, b_i, c_i, d_i . For perfect reconstruction the values \tilde{g}_{ℓ} just have to satisfy the PR constraint given in (10), resulting in three additional free variables. However, for the filter bank to be free of DC leakage, the final polyphase filter $G_{\ell}(z)$ must satisfy (9) which determines \tilde{g}_{ℓ} as shown in (24). We still have to verify that these values \tilde{g}_{ℓ} also satisfy the PR constraint in (10). Multiplying the upper and the lower equation in (24) we obtain:

$$\begin{aligned} &\tilde{g}_{\ell} \tilde{g}_{2M-1-\ell} + \tilde{g}_{M+\ell} \tilde{g}_{M-1-\ell} \\ &= (\alpha_{\ell} \alpha_{2M-1-\ell} + \alpha_{M+\ell} \alpha_{M-1-\ell}) (-1)^s \end{aligned} \quad (36)$$

and thus with (8) the PR constraint (10).

$$\begin{aligned}
[G_\ell(z) G_{M+\ell}(z)] &= [\tilde{g}_\ell \tilde{g}_{\ell+M}] \prod_{j=1}^{j_{\max}} \begin{bmatrix} 1 & 0 \\ a_j z^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & b_j \\ 0 & 1 \end{bmatrix} \prod_{i=1}^{i_{\max}} \begin{bmatrix} z^{-1} & 0 \\ c_i & 1 \end{bmatrix} \begin{bmatrix} 1 & d_i \\ 0 & z^{-1} \end{bmatrix} \prod_{l=1}^{l_{\max}} \begin{bmatrix} 1 & 0 \\ a_{0,l} z^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_{0,l} \\ 0 & z^{-1} \end{bmatrix} \\
\begin{bmatrix} G_{2M-1-\ell}(z) \\ G_{M-1-\ell}(z) \end{bmatrix} &= \prod_{l=1}^{l_{\max}} \begin{bmatrix} z^{-1} & -d_{0,l} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -a_{0,l} z^{-1} & 1 \end{bmatrix} \prod_{i=1}^{i_{\max}} \begin{bmatrix} z^{-1} & -d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c_i & z^{-1} \end{bmatrix} \prod_{j=1}^{j_{\max}} \begin{bmatrix} 1 & -b_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -a_j z^{-1} & 1 \end{bmatrix} \begin{bmatrix} \tilde{g}_{2M-1-\ell} \\ \tilde{g}_{M-1-\ell} \end{bmatrix}
\end{aligned} \tag{23}$$

with $m = i_{\max} + j_{\max} + l_{\max} + 1$, $s = l_{\max} + 2i_{\max}$, $l_{\max} \in \{0, 1\}$

$$\begin{aligned}
[\tilde{g}_\ell \tilde{g}_{\ell+M}] &= [\alpha_\ell \alpha_{\ell+M}] \left(\prod_{j=1}^{j_{\max}} \begin{bmatrix} 1 & 0 \\ -a_j & 1 \end{bmatrix} \begin{bmatrix} 1 & b_j \\ 0 & 1 \end{bmatrix} \prod_{i=1}^{i_{\max}} \begin{bmatrix} -1 & 0 \\ c_i & 1 \end{bmatrix} \begin{bmatrix} 1 & d_i \\ 0 & -1 \end{bmatrix} \prod_{l=1}^{l_{\max}} \begin{bmatrix} 1 & 0 \\ -a_{0,l} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_{0,l} \\ 0 & -1 \end{bmatrix} \right)^{-1} \\
\begin{bmatrix} \tilde{g}_{2M-1-\ell} \\ \tilde{g}_{M-1-\ell} \end{bmatrix} &= \left(\prod_{l=1}^{l_{\max}} \begin{bmatrix} -1 & -d_{0,l} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a_{0,l} & 1 \end{bmatrix} \prod_{i=1}^{i_{\max}} \begin{bmatrix} -1 & -d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c_i & -1 \end{bmatrix} \prod_{j=1}^{j_{\max}} \begin{bmatrix} 1 & -b_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a_j z^{-1} & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} \alpha_{2M-1-\ell}(z) \\ \alpha_{M-1-\ell}(z) \end{bmatrix}
\end{aligned} \tag{24}$$

with $\alpha_\ell, \alpha_{\ell+M}, \alpha_{M-1-\ell}$, and $\alpha_{2M-1-\ell}$ according to (9)

4. DESIGN EXAMPLE

Figure 3 shows the analysis filters magnitude frequency responses for a 4-channel cosine-modulated filter banks obtained by non-linear optimization of (23) and (24). The filter length is $N = 32$ and the system delay $D = 31$. All analysis filters apart from the lowpass have a zero at frequency zero.

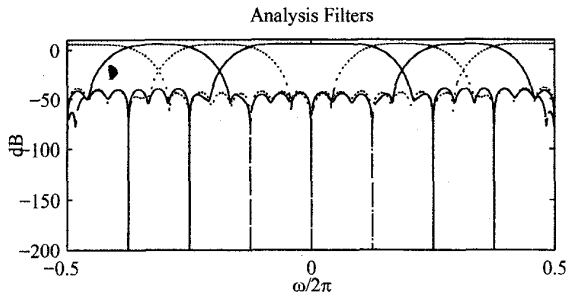


Figure 3: Analysis filters of a 4-channel cosine-modulated filter bank without DC leakage

5. CONCLUSIONS

In this paper we have presented an implementation for the prototype filters of biorthogonal cosine-modulated filter banks that automatically guarantees PR of the bank as well as a zero of all analysis filters apart from the lowpass filter at frequency zero. It is based on the lifting scheme and in order to obtain no DC leakage just the values \tilde{g}_ℓ of the first lifting step have to be fixed. The frequency response of the prototype can be optimized using non-linear optimization methods. The coefficients are robust to quantization and the proposed implementation can also be used in order to design integer coefficient prototypes.

6. REFERENCES

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