Methods

One MRI k-space measurement is composed of several partial measurements $\mathbf{y}_n$ along specified trajectories within one readout per different time interval $n$. Patient motion within one readout is assumed to be constant due to small intervals. A suitable image reconstruction problem is given by

$$\mathbf{x}_0 = \text{arg} \min_{\mathbf{x}} \sum_{n=1}^{N} \mathcal{D}_n \mathbf{F} \mathbf{x} - \mathbf{y}_n^1 + \frac{1}{2} \mathbf{x}^T \mathbf{D}_n \mathbf{x}.$$ 

with the sampling operator $\mathcal{S}_n$, representing the trajectory at time $n$, $\mathbf{F}$, sensing the Fourier transform of the image $\mathbf{x}$ which is a motion corrupted by operator $\mathbf{D}_n$.

Sampling is described by $\mathbf{D}_n$ on arbitrary k-space trajectories and needs the computation with real-valued nonequidistant frequency coordinates. It is calculated by the nonperiodic discrete Fourier transform (NDFT) [9].

Two motion models are combined in $\mathbf{D}_n$ for translation and rotation separately. Full and pixel level translation shifts are mathematically modeled by convolution matrices in image space [9]. Rotation is described by applying a rotation matrix to the sampling trajectory coordinates followed by a barycentric interpolation [10] combined with Delaunay triangulation [11] of the coordinates. This delivers a differentiable motion model on arbitrary nonperiodic frequency coordinates.

A meaningful regularization on motion corrupted MRI images is enforcing sparsity in the wavelet domain by $\mathbf{D}_n [\mathbf{W} \mathbf{x}]$, to give advantage to clear natural structures.

For motion estimation, a three-step iteration algorithm is performed. A sparsifying ADMM [12] is evaluated in the first step. It solves the minimization for fixed motion parameters by splitting it into a measurement fitting and a sparsifying problem which are iteratively solved and updated by

$$\mathbf{x}_i = \text{arg} \min_{\mathbf{x}} \sum_{n=1}^{N} \mathcal{D}_n \mathbf{F} \mathbf{x} - \mathbf{y}_n^1 + \frac{1}{2} \mathbf{x}^T \mathbf{D}_n \mathbf{x} + \mathbf{u}_n^1 - \mathbf{u}_n^2.$$ 

The measurement fitting problem is solved by conjugate gradients [2] using the inverse NDFT [9]. The sparsity of the solution for the second problem is enforced by thresholding in the wavelet domain. In the second step, motion estimation is achieved by the sparsified reconstruction of the ADMM in the derivatives of the motion models with Newton’s gradient method [13]. Thereby, the parametrization of translation is improved with a Gaussian model. The last step updates the globally estimated motion on the k-space frequency coordinates and coefficients.

Finally, image reconstruction is performed by gridding [5] to overcome blurring of the inverse NDFT. The frequency coefficients are reassembled onto a Cartesian grid. As the arbitrary trajectory coordinates were not equally dense over the whole k-space, the coefficients are convolved with an area density compensation function. This weighting is computationally efficient calculated by partitioning the k-space into rectangles around the sampling points and saving the structure in k-trees [6].

Results

Representatively, PROPPELAR and radial trajectories were simulated. The algorithm was tested on BrainWeb [7] and Shepp-Logan phantom data in a FOV of 455 and 160 pixels, respectively. Smooth patient motion was modeled as an autoregressive moving average process with maximum translation amplitudes up to 30 pixels and maximum rotation up to 45°.

To balance motion corrupted images and the motion compensated results of the algorithm calculated with PROPPELAR trajectories. In the corrupted images, no structure or even no image is identifiable. The reconstructed images do not show motion artefacts, but clear contours and details are visible. Only a few gridding artefacts appear due to low resolution. In Figure 2, the progresses of the corresponding originally applied and by the algorithm estimated motion are shown. Their courses follow very closely. The table in Figure 3 shows the mean percentage improvement of the PSNR and mutual information (MI) between motion corrupted images and motion compensated reconstructions for different maximum translation and rotation amplitudes.

Discussion

The results prove the ability of the algorithm to estimate rigid motion very exactly. Even from images corrupted with motion shifts about one 8th of the FOV very detailed images are reconstructed. Small differences between the original and estimated motion just shift the position of the image centroid but do not effect the image quality. The high improvements of the image quality measures emphasize the impression given by the reconstructed images. The method is easily adaptable to other motion models and with it expandable to physical descriptions of elastic motion. As sampling is mathematically described for real-valued nonequidistant frequency coordinates the algorithm quality is largely independent of the design of MRI trajectories.

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References

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