PROCESSING FINITE-LENGTH SIGNALS WITH MDFT FILTER BANKS

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RÉSUMÉ

Plusieurs publications récentes présentent des méthodes qui permettent d'obtenir des bancs de filtres à reconstruction parfaite préservant le support temporel de signaux d'entrée de longueur finie. Ces méthodes utilisent une extension temporelle de ces signaux [1, 2], et un design de filtres de transition pour les bords [3, 4, 5]. Elles ne peuvent cependant pas être appliquées directement aux bancs de filtres modulés et à phase linéaires dont les centres de symétrie des filtres d'analyse sont différents [6, 7, 8, 9]. Néanmoins, en prolongeant le signal d'entrée de manière appropriée, on peut retrouver les propriétés de reconstruction parfaite et de préservation du support temporel fini avec de tels bancs de filtres.

Dans cet article, nous nous concentrons sur des méthodes qui utilisent l'extension du support de signaux finis, appliquées aux bancs de filtres MDFT (ceux-ci peuvent être considérés comme un cas général pour la classe des bancs de filtres étudiés). Les résultats ainsi obtenus peuvent être directement utilisés dans les bancs de filtres proposés par Princen et Bradley [6] et par Lin et Vaidyanathan [9].

ABSTRACT

Several recent publications treat extension methods for finite-length input signals [1, 2] and the design of boundary filters [3, 4, 5], respectively, leading to support preservative and perfect reconstruction filter banks. These methods cannot be applied in their original form to modulated linear phase filter banks where the analysis filters have different centers of symmetry [6, 7, 8, 9]. However, these filter banks can keep the support preservative and perfect reconstruction property, too, if we appropriately extend the input signal.

In this paper we only treat extension methods for the MDFT filter bank as it can be regarded as the most general case of the considered class. The results can be directly transferred to the filter banks proposed by Princen and Bradley [6] and Lin and Vaidyanathan [9].

1 INTRODUCTION

The aim of subband coding is to compress the input signal by using appropriate quantization and coding techniques in the subbands. In order to achieve a high compression ratio it is important that the total number of subband samples is not greater than the number of input samples.

When processing infinite-length input sequences with a modulated M channel filter bank, critical subsampling is obtained if each of the M subbands is decimated by the factor M.

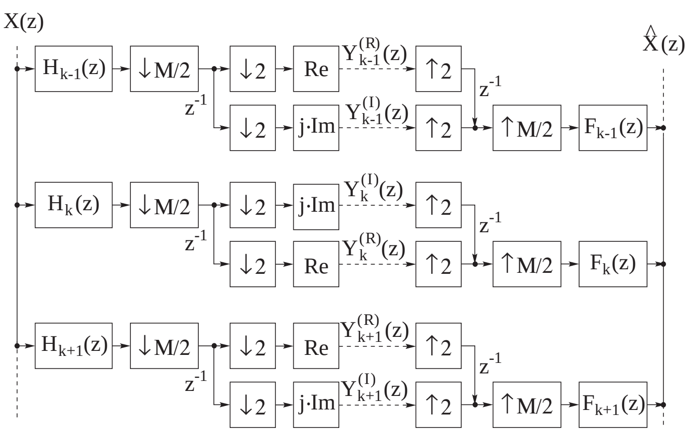

Fig. 1: Modified complex modulated filter bank

The MDFT filter bank given in Fig. 1 is a modified complex modulated M channel filter bank providing perfect reconstruction [8] and critical subsampling for infinite-length input signals. M complex valued input samples yield M purely real and M purely imaginary valued subband signals. For real valued input signals, M of the $2M$ subband signals are complex conjugates of the others and can be removed. The MDFT filter bank can be implemented efficiently by a DFT polyphase realization [10].

The causal analysis and synthesis filters $h_k(n)$ and $f_k(n)$ are complex modulated and time shifted versions of a zero-phase lowpass prototype $p(n)$ with a transition bandwidth from $-\pi/M$ to π/M. As shown in [8], all analysis and synthesis filters are linear phase and given by

$$h_k(n) = f_k(n) = p(n-\frac{N-1}{2}) \exp\left(j\frac{2\pi k(n-(N-1)/2)}{M}\right),$$
$$n = 0, \dots N-1, \quad k = 0, \dots, M-1. \qquad (1)$$

Concerning the linear phase property, the MDFT filter bank differs from cosine modulated filter banks [11, 12] which use a linear phase prototype but do no have linear phase analysis / synthesis filters.

When $x(n)$ is a finite-length input signal, due to the linear convolution of $x(n)$ with the analysis filters $h_k(n)$, the number of real and imaginary valued samples in all subbands is longer than twice the length of the complex valued input signal.

Several possibilities to reduce the data rate in the subbands exist:

- The easiest way is to truncate the subband signals. This leads to distortions in the boundaries and perfect reconstruction cannot be obtained.
- Another possibility is to use circular convolution [13]. The input sequence can be interpreted as a period of a longer periodic signal. The subband signals are also periodic and one period contains all information necessary for perfect reconstruction. By this, we can obtain critical subsampling. The main disadvantage of this method is the fact that the amplitude of the periodically extended signal may contain discontinuities. This causes a few samples with high amplitudes in the higher frequency bands. Since the average energy in these bands is generally very low, only a few bits are spent for their quantization so that high quantization errors may be visible in the boundary regions of the reconstructed signal.
- For filter banks with linear phase analysis / synthesis filters it is possible to extend the input sequence symmetrically and to obtain symmetrically extended subband signals [1, 2]. Discontinuities of the signal's amplitude are avoided by this method. The subband signals can be truncated in such a way that the filter bank is support preservative and provides perfect reconstruction. However, the symmetries described in [2] are only valid if all analysis filters have the same center of symmetry and therefore symmetric extension methods cannot be applied in their original form to the MDFT filter bank and the filter banks proposed by Princen and Bradley [6] and Lin and Vaidyanathan [9].
- Apart from extension methods of the input signal it is possible to design boundary and transition filters, respectively, [3, 4, 5]. These filters are no longer modulated versions of a common prototype and the MDFT filter bank cannot be implemented any longer in the DFT polyphase realization when using these filters.

Outline of the paper: In section 2 we will show how the symmetric extension method can be applied to MDFT filter banks by having a closer look at the symmetries appearing in subbands decimated without and with a delay for a one-dimensional complex valued input signal. In section 3 we will apply the results to a real valued AR1 process and to an image. The subband signals of the image will be quantized and the visible boundary distortions will be compared to those of simple truncation and circular convolution, respectively.

2 SYMMETRIC EXTENSION OF THE INPUT SIGNAL

In this paper we will regard even lengths analysis and synthesis filters and input signals whose lengths are integer multiples of M. For the sake of simplicity, the signal length $N_x = mM$ $(m > 0)$ is assumed to be greater than the duration of the filter impulse responses.

Instead of the input signal $x(n)$ we will process the extended signal $x_{ex}(n)$

$$\mathbf{x}_{ex} = \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{I}_{N_x} \\ \mathbf{E}_2 \end{bmatrix} \mathbf{x} \quad \text{with} \quad \mathbf{x} = [x(0), x(1), \ldots, x(N_x-1)]^T$$

where $\mathbf{I}_{N_x}$ denotes the size $N_x \times N_x$ identity matrix and $\mathbf{E}_1$ and $\mathbf{E}_2$ the size $k \times N_x$ reflexion matrices with $k = N/2+1+\lambda$ and $\lambda \geq 0$.

The matrices $\mathbf{E}_1$ and $\mathbf{E}_2$ are

$$\begin{aligned} [\mathbf{E}_1]_{i,j} &= \delta_{k-1-i,j} \\ [\mathbf{E}_2]_{i,j} &= \delta_{i,N_x-1-j} \end{aligned} \quad i = 0, \ldots, k-1, \; j = 0, \ldots N_x - 1$$

with $\delta_{i,j}$ being the Kronecker symbol.

Fig. 2 shows an example for the computation of the decimated lowpass subband signal from an extended input signal in a 4 channel MDFT filter bank. The impulse response of the lowpass filter $h(n)$ has the length $N = 6$ and the input signal $x(n)$ the length $N_x = 8$. The symmetric extension is shown in dashed boxes. During convolution the values of $x(n)$ and $h(-n)$ are multiplied and accumulated for the delays shown in Fig. 2.

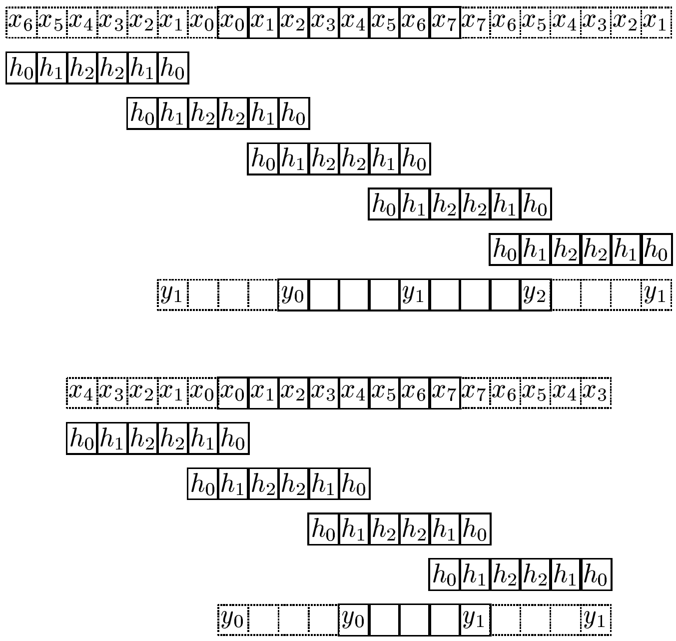

Fig. 2: Symmetries of the lowpass subband signal due to convolution with the extended input signal and subsampling without and with a delay

From Fig. 2 we see that symmetric subband signals $y(m)$ are formed in both examples. In the upper one, the subband signal has 3 non-redundant samples (y_0, y_1, y_2). This does not correspond to critical subsampling where only $N_x/M = 8/4 = 2$ non-redundant samples may appear. This is the case in the lower example.

Due to subsampling with and without a time delay of $M/2$ samples both kinds of symmetry exist in the subbands of the MDFT filter bank. Nevertheless, it is possible to obtain perfect reconstruction and support preservation when we regard all subbands.

$$
\mathbf{Y}_1 = \left[\begin{array}{ccccccccc}
\cdots & y_0^R(1) & y_0^R(0) & y_0^R(1) & \cdots & y_0^R(m-1) & y_0^R(m) & y_0^R(m-1) & \cdots \\
 & -y^I_{M-1}(1) & y_1^I(0) & y_1^I(1) & & y_1^I(m-1) & y_1^I(m) & -y^I_{M-1}(m-1) & \\
\cdots & \vdots & \vdots & \vdots & & \vdots & \vdots & -y^R_{M-2}(m-1) & \cdots \\
 & & y^I_{M/2-1}(0) & y^I_{M/2-1}(1) & & y^I_{M/2-1}(m-1) & y^I_{M/2-1}(m) & \vdots & \\
 & -y^R_{M/2}(1) & 0 & y^R_{M/2}(1) & \cdots & y^R_{M/2}(m-1) & 0 & -y^R_{M/2}(m-1) & \\
\cdots & \vdots & -y^I_{M/2-1}(0) & & & y^I_{M/2+1}(m-1) & -y^I_{M/2-1}(m) & \vdots & \cdots \\
 & & \vdots & \vdots & & \vdots & & & \\
 & -y_1^I(1) & -y_1^I(0) & y^I_{M-1}(1) & \cdots & y^I_{M-1}(m-1) & -y_1^I(m) & -y_1^I(m-1) &
\end{array}\right] \quad (2)
$$

$$
\mathbf{Y}_2 = \left[\begin{array}{cccccccc}
\cdots & y_0^I(0) & y_0^I(0) & y_0^I(1) & \cdots & y_0^I(m-1) & y_0^I(m-1) & \cdots \\
 & -y^R_{M-1}(0) & y_1^R(0) & y_1^R(1) & & y_1^R(m-1) & -y^R_{M-1}(m-1) & \\
 & & \vdots & \vdots & & \vdots & & \\
\cdots & -y_2^I(0) & y^I_{M-2}(0) & y^I_{M-2}(1) & & y^I_{M-2}(m-1) & -y_2^I(m-1) & \cdots \\
 & -y_1^R(0) & y^R_{M-1}(0) & y^R_{M-1}(1) & \cdots & y^R_{M-1}(m-1) & -y_1^R(m-1) &
\end{array}\right] \quad (3)
$$

For an input signal of the length $N_x = mM$, the matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$ in Eqns. (2) and (3) contain the subband signals decimated without and with a delay, respectively. The k-th row contains the k-th subband signal. The real and imaginary part has been taken corresponding to the filter bank structure (see Fig. 1) starting with the real part in the undelayed branch of the lowpass subband.

In Eqns. (2) and (3), the non-redundant subband samples that must be saved in order to obtain perfect reconstruction are shown in dashed boxes. For synthesis, the samples outside the boxes have to be extended.

The zeroth row of $\mathbf{Y}_1$ has the same symmetry as the upper example in Fig. 2 and the symmetry in the zeroth row of $\mathbf{Y}_2$ is identical to the lower example in Fig. 2. In the other rows the symmetric extension is done in the following way: the k-th row ($k = 1, \ldots, M-1$) has to be filled up with the flipped negative samples of the $(M-k)$-th row. In the highpass band in matrix $\mathbf{Y}_1$ ($M/2$-th row) the zeroth and m-th sample are equal to zero. These unusual symmetries are due to the relation between the analysis filters $h_k(n)$ and $h_{M-k}(n)$:

$$
h_k(n) = -h^*_{M-k}(n), \quad k = 1, \ldots, M-1 \qquad (4)
$$

Each subband signal in $\mathbf{Y}_2$ (i.e. each row) has m non-redundant samples. In $\mathbf{Y}_1$, $M/2$ subbands contain $(m+1)$ non-redundant samples and the remaining $M/2$ subbands only $(m-1)$. Overall, we have fulfilled the support preservative property (the complex valued input signal of the length $N_x = mM$ is devided into $2M$ purely imaginary or real valued subband signals. M subbands of the length m, $M/2$ subbands of the length $(m+1)$, and $M/2$ subbands of the length $(m-1)$).

3 RESULTS

A. Application to a Real Valued AR1 Process

When using symmetric extension methods we have ensure that the statistics of the subband signals remain nearly

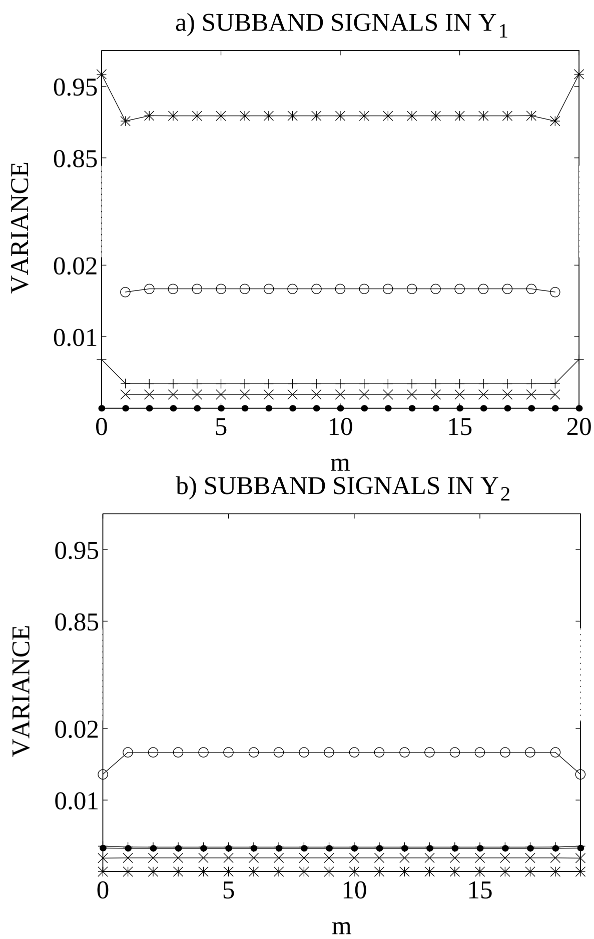

Fig. 4: Variance of the subband signals; $y_0(m)$: ***; $y_1(m)$: ooo; $y_2(m)$: +++; $y_3(m)$: xxx; $y_4(m)$: ...; $y_k(m) = y_{8-k}(m)$, $k = 5, \ldots, 7$

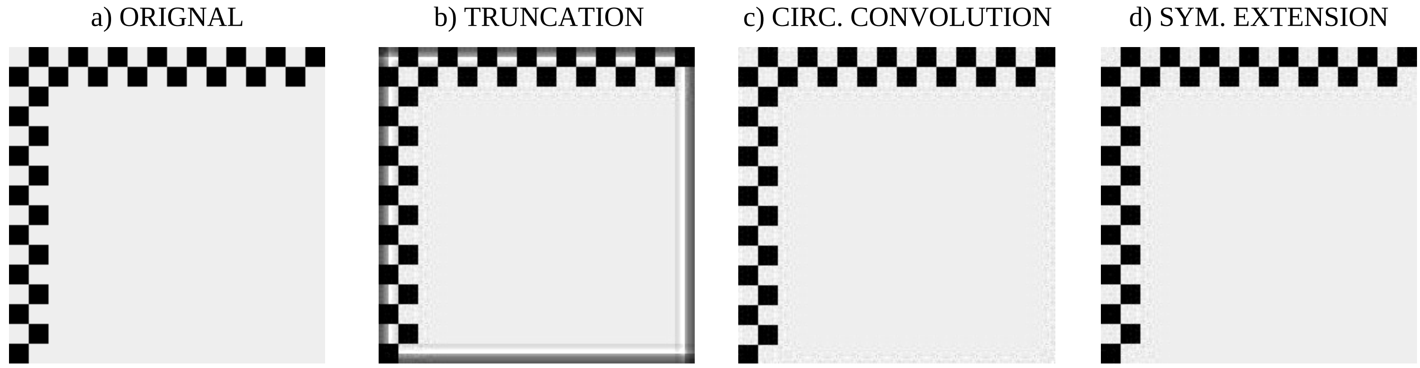

Fig. 3: Influence of different boundary treatment of the quantized subbands: a) original, b) truncation of the subbands, c) circular convolution, d) symmetric extension

unchanged in order to avoid boundary distortions. Fig. 4 shows the variances of the subband signals for a real valued AR1 lowpass process with $\rho = 0.95$ using an 8 channel filter bank. The analysis filters have $N = 40$ coefficients and the input signal has $N_x = 160$ samples. The variances hardly change at the boundaries.

The lowpass band $y_0(m)$ and the highpass band $y_4(m)$ being decimated with and without a delay, respectively, are equal to zero because of a real valued input signal.

B. Application to Image Coding

Images are typical examples of finite-length input signals. In this subsection we show the influence of different treatments of the boundaries. The original 256×256 pixel image is shown in Fig. 3a. It is devided into subbands by processing the rows and columns separately using a 16 channel MDFT filter bank and a lowpass prototype with 32 coefficients. All subbands are linearly quantized with the same quantization step.

The boundary distortions visible in the reconstructed image are worst when the subbands are just truncated to the size of the original image, see Fig. 3b. When using circular convolution visible boundary distortions appear in the lower and right border of the reconstructed image in Fig. 3c due to the discontinuities introduced by this method. Using symmetric extension methods hardly any boundary distortions are visible in the reconstructed image, see Fig. 3d.

In all reconstructed images so called “ringing effects” appear, too, which are due to the quantization error in the subbands.

4 CONCLUSION

In this paper, we have shown how extension methods can be applied to MDFT filter banks. These filter banks can be regarded as a general case of linear phase filter banks having analysis filters with different centers of symmetry. An example has shown that the boundary distortions are significantly reduced in comparison to circular convolution and simple truncation of the subband signals.

REFERENCES

[1] M. J. T. Smith and S. L. Eddins. Analysis/Synthesis Techniques for Subband Image Coding. *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-38:1446–1456, August 1990.

[2] L. Chen, T. Nguyen, and K. P. Chan. Symmetric Extension Methods for M-Channel PR LP FIR Analysis/Synthesis Systems. In *Proc. ISCAS*, London, UK, June 1994.

[3] R.L. de Queiroz. Subband Processing of Finite Length Signals without Border Distortions. In *Proc. IEEE ICASSP*, volume 4, pages 613–616, 1992.

[4] C. Herley and M. Vetterli. Orthogonal Time-Varying Filter Banks and Wavelet Packages. *IEEE Trans. on Signal Processing*, SP-42:2650–2663, Oct. 1994.

[5] A. Mertins. Time-Varying and Support Preservative Filter Banks: Design of Optimal Transition and Boundary Filters via SVD. In *Proc. ICASSP*, Detroit, USA, May 1995.

[6] J. P. Princen and A. B. Bradley. Analysis / Synthesis Filter Bank Design Based on Time Domain Aliasing Cancellation. *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-34:1153–1161, October 1986.

[7] N. J. Fliege. Computational Efficiency of Modified DFT-Polyphase filter banks. In *Proc. 27th Asilomar Conference on Signals, Systems and Computers*, Asilomar, USA, Nov. 1993.

[8] T. Karp and N. J. Fliege. MDFT Filter Banks with Perfect Reconstruction. In *Proc. ISCAS*, Seattle, USA, May 1995.

[9] Y. Lin and P. P. Vaidyanathan. Linear Phase Cosine Modulated Maximally Decimated Filter Banks with Perfect Reconstruction. In *Proc. ISCAS*, volume 2, pages 17–19, London, UK, June 1994.

[10] N. J. Fliege. *Multirate Digital Signal Processing.* John Wiley and Sons, Chichester, UK, 1994.

[11] R. D. Koilpillai and P. P. Vaidyanathan. Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction. *IEEE Trans. on Signal Processing*, SP-40:770–783, April 1992.

[12] P. P. Vaidyanathan. *Multirate Systems and Filter Banks.* Prentice Hall, Englewood Cliffs, 1993.

[13] J. Woods and S. O'Neil. Subband Coding of Images. *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-34:1278–1288, October 1986.