MMSE MPI Reconstruction Using Background Identification

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INTRODUCTION Magnetic particle imaging (MPI) is an imaging method for the determination of the distribution of superparamagnetic iron oxide nanoparticles (SPIONs) injected into the human body [1]. Since the SPIONs often do not spread within the whole field of view (FOV) due to anatomical structures, areas without SPIONs are likely to exist in the FOV. As each column of the system matrix represents the impact of one position in the FOV, we propose to estimate which columns of the system matrix correspond to the positions in the FOV without SPIONs. The advantage of identifying and removing the columns of the system matrix belonging to positions without particle concentration is studied. Several methods to find these positions are introduced and evaluated. Furthermore, a method for reconstruction based on the minimum mean-square error (MMSE) estimator is applied.

MATERIAL AND METHODS The particle concentration $\boldsymbol{c}$ is reconstructed from the voltage signal $\boldsymbol{u}$ with the help of the system matrix $\boldsymbol{S}$ by the signal equation $\boldsymbol{u} = \boldsymbol{S}\boldsymbol{c}$, where $\boldsymbol{u} \in \mathbb{R}^N$, $\boldsymbol{S} \in \mathbb{R}^{N\times M}$ and $\boldsymbol{c} \in \mathbb{R}_+^M$. The reconstruction based on the MMSE estimator uses $\hat{\boldsymbol{c}}(\boldsymbol{u}) = \left[\boldsymbol{R_{cc}}^{-1} + \boldsymbol{S^T R_{nn}^{-1} S}\right]^{-1} \boldsymbol{S^T R_{nn}^{-1} u}$, where $\boldsymbol{R_{cc}} \in \mathbb{R}^{M\times M}$ represents a matrix used for regularization and $\boldsymbol{R_{nn}} \in \mathbb{R}^{N\times N}$ is the correlation matrix of additive zero-mean noise $\boldsymbol{n}$ [2]. Here, white Gaussian noise is assumed, which leads to $\boldsymbol{R_{nn}} = \boldsymbol{\sigma_n^2 I}$. The matrix $\boldsymbol{R_{cc}} = \boldsymbol{E}\{\boldsymbol{cc^T}\}$ is the correlation matrix for the particle concentration $\boldsymbol{c}$. It can be generated based on a priori knowledge of the typical particle concentrations. The applied system matrix $\boldsymbol{S}$ is obtained by simulation of a Lissajous sequence [3].

To identify background pixels with the SPION concentration zero, an iterative threshold method and a linear classifier are investigated. For the experiments, phantoms showing different lower case letters with the values zero (background) and one (foreground) were designed and divided up into training and test data. With the knowledge of the identified background pixels, the corresponding columns of the system matrix can be eliminated in the signal equation.

The iterative thresholding method detects pixels with the value zero in several iterations. In each iteration, a reconstruction based on the MMSE estimator is performed, followed by an application of a threshold value deciding which pixels are assigned with the value zero. Afterwards, the corresponding columns of the system matrix are deleted and the modified system matrix is used for the reconstruction in the following iteration. The algorithm terminates as soon as the threshold value stops identifying pixels with the value zero.

The linear classifier identifies pixels with the value zero based on its neighborhood. By applying Fisher's linear discriminant analysis, a linear function that allows us to differentiate between pixels of the background and foreground is determined.

As a method to prevent pixels of the foreground to be classified as background, the classification is followed by a correction step that assigns every pixel in a four-pixel neighborhood of a foreground pixel as foreground as well.

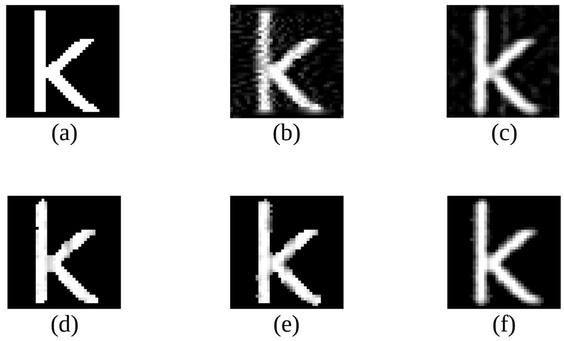

Figure 1: Phantom (a) and reconstruction results (b-f) for SNR=30dB.

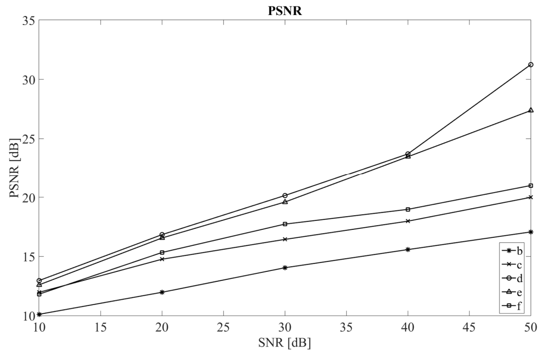

Figure 2: PSNR for SNR of 10-50dB.

RESULTS Fig. 1 shows a comparison of the reconstruction results for different cases. (b) MMSE estimator with $\boldsymbol{R_{cc_\lambda}} = \lambda \boldsymbol{I}$; (c) MMSE estimator with proper $\boldsymbol{R_{cc}}$; (d) MMSE estimation after background pixel identification by iterative thresholding method and deletion of columns of the system matrix; (e) MMSE estimation after background-pixel deletion based on a linear classifier; (f) MMSE estimation after background deletion based on a linear classifier combined with post processing. As can be seen in Fig. 2, the peak signal-to-noise ratio (PSNR) of (d-f) with identified background pixels is higher than the PSNR of (b-c) without identified background pixels.

CONCLUSION We were able to show that a proper correlation model $\boldsymbol{R_{cc}}$ improves the reconstruction result. Furthermore, removing the columns of the system matrix corresponding to background pixels identified by our introduced methods can improve the result.

REFERENCES

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