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Graphical stochastic models for tracking applications with variational message passing inference

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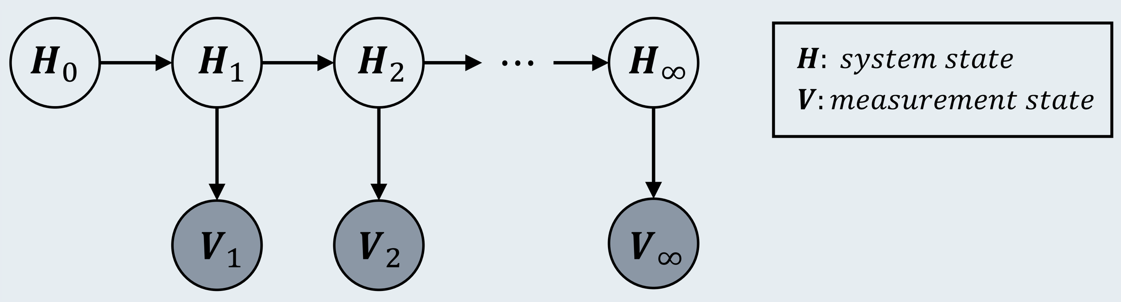
1. Introduction

- **Motivation:**
  State estimation filter:
  - Often the dynamic behaviour of a system is most accurately described as a **hidden Markov process**.
  - The estimation of the hidden interal states for such systems, based on observations, requires the application of a **state estimation filter**.

  Weakness and Limitations:
  - State-of-the-art estimation filters, such as the **Kalman filter**, its derivates, or the **particle filter**, have various weaknesses in system environments with **nonlinear process dynamics** and **non-Gaussian noise**.
- **Our research:**
  - We developed a novel filter approach to the problem of state estimation in the context of hidden Markov processes and non-Gaussian noise within the framework of **graphical stochastical models** as well as **variational message passing**.
  - Our experiments show that the proposed method provides notable **qualitative and numerical advantages** over state-of-the-art methods in multiple practical and challenging scenarios.

2. VMP inference concept

- **State estimation filter:**
  - If a dynamic process is accurately described by a generative hidden Markov model, it follows this stochastic-graphical structure:



The joint distribution of such a process is given by

$$P(\boldsymbol{H}_{0:k}, \boldsymbol{V}_{1:k}) = P(\boldsymbol{H}_0) \cdot \prod_{i=1}^{k} P(\boldsymbol{H}_i|\boldsymbol{H}_{i-1}) \cdot P(\boldsymbol{V}_i|\boldsymbol{H}_i) \ .$$

  - If noise within such a process is not zero-mean Gaussian distributed, then the solution for the state estimation problem $P(\boldsymbol{H}_k|\boldsymbol{V}_{1:k})$, can not be derived in a closed-analytical form.
  - As a solution to this problem, we propose a filter approach within the framework of graphical stochastical models as well as variational message passing.
- **Variational message passing (VMP):**
  - VMP, proposed by Winn et al.([1]), is a deterministic, iterative inference method to approximate an unknown likelihood function $P(\boldsymbol{H}|\boldsymbol{V})$ by a tractable function $Q(\boldsymbol{H})$, according to:

$$Q_{opt}(\boldsymbol{H}) \approx P(\boldsymbol{H}|\boldsymbol{V}) \,.$$

In detail, the approximation is achived by solving the optimization problem

$$Q_{opt}(\boldsymbol{H}) = \underset{Q}{\operatorname{argmax}}(\mathcal{L}(Q)) = \underset{Q}{\operatorname{argmax}}\left(\sum_{\boldsymbol{H}} \ln\left(\frac{P(\boldsymbol{H},\boldsymbol{V})}{Q(\boldsymbol{H})}\right) \cdot Q(\boldsymbol{H})\right) \ ,$$

under the constraints of:
  - **Acyclic graph structure:** The stochastic process has to be accurately modeled by an acyclic graphical Bayesian network.
  - **Factorisation:** The solution space of $Q(\boldsymbol{H})$ is restricted to

$$Q(\boldsymbol{H}) = \prod_i Q_i(\boldsymbol{H}_i) \ .$$

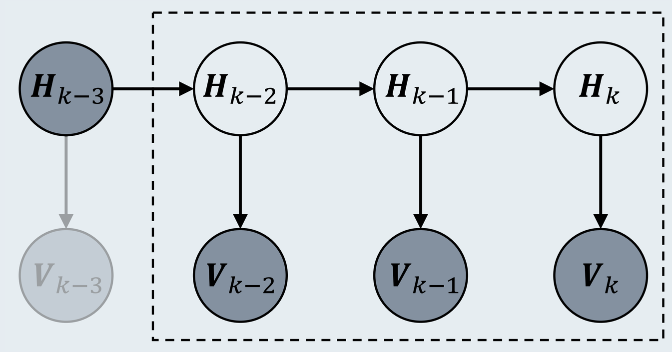
  - **Exponential Family:** Each likelihood $P(\boldsymbol{X}_i|Pa(\boldsymbol{X}_i))$ is a function of the exponential family.

3. Iterative formulation of a VMP based estimation filter

- For a dynamic hidden Markov process, which fullfils the above listed VMP restrictions, VMP performs an approximative inference of the most probable realisation of $P(\boldsymbol{H}_k|\boldsymbol{V}_{1:k})$ in the form of

$$\tilde{\boldsymbol{H}}_k = \langle P(\boldsymbol{H}_k|\boldsymbol{V}_{1:k})\rangle = \langle Q_k(\boldsymbol{H}_k)\rangle \,.$$

- Over time, a naïve VMP implementation would cause numerical issues. Therefore, we propose the introduction of a time-history-limiting **sliding-window concept**.

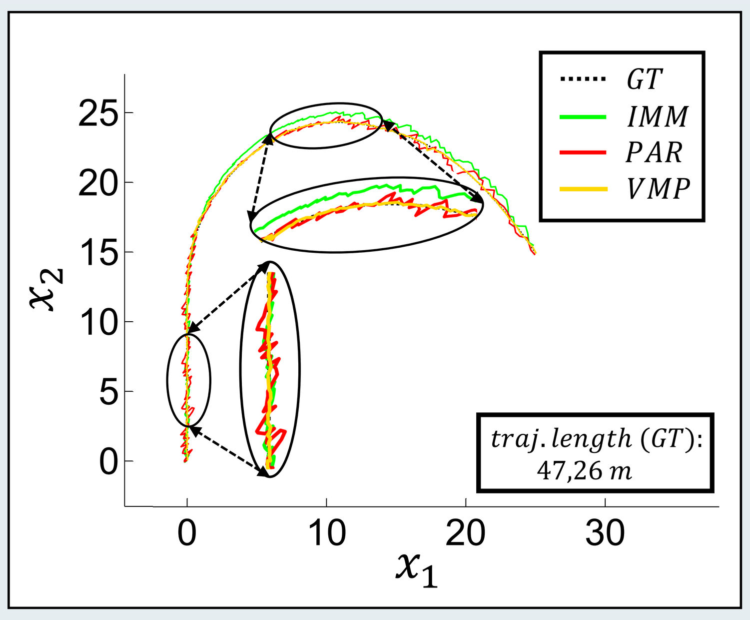


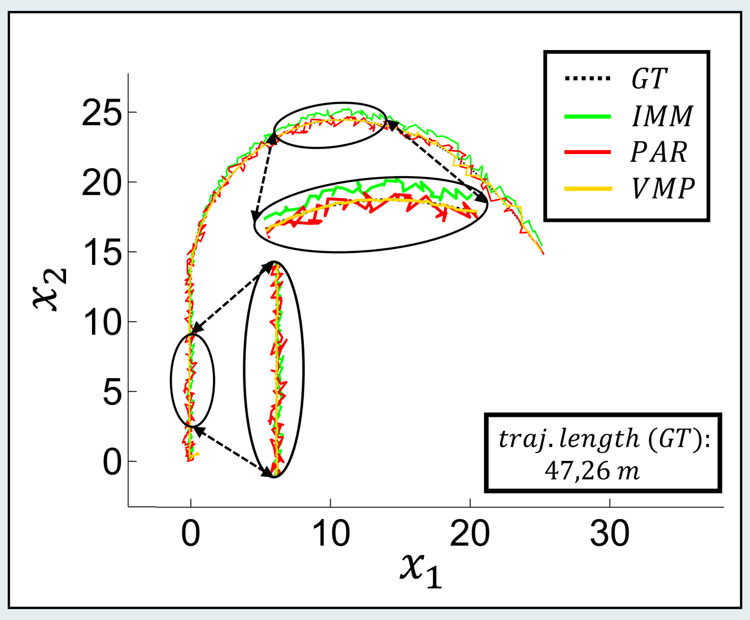
- In order to receive a numerically efficient implemention, we recommend to choose the **sliding-window size**, with regard to the characteristics of the process and the estimation accuracy, as **small as possible**.

4. Results

We evaluate the performance of the proposed filter approach by comparing it with various state-of-the-art methods (IMM[3], PAR[4]) in challenging tracking scenarios.

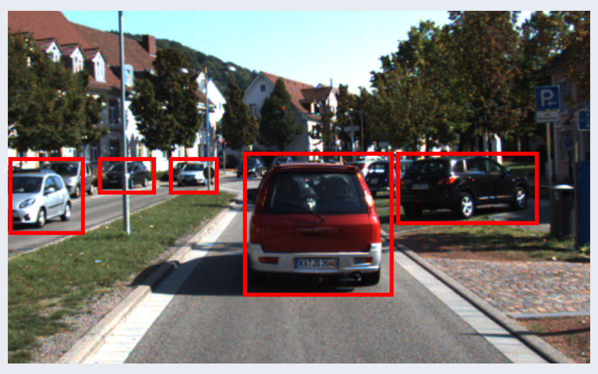
- **Synthetic data:**
  - Simulative 2D trajectory estimation of a CWPA([2])-model-moved object.
  - The measurement signals of the external position sensor are assumed as superimposed by non-Gaussian noise





quality [time]	IMM	PAR	VMP (our method)
Gaussian-noise	1.1 [1]	1.1 [202.4]	**1** [23.7]
GMM-noise (**w**)	4.4 [1]	2.0 [245.8]	**1** [38.2]
GMM-noise (**s**)	4.1 [1]	1.5 [275.1]	**1** [54.3]

- **KITTI dataset : Tracking of classifer-detected object bounding boxes**
  - The positions of LSVM-detected 2D object bounding boxes are improved by a state estimation filter (evaluated KITTI dataset ([5]) spectrum: 2948 frames, 78 different object tracks).
  - The LSVM-raw detections are superimposed by non-Gaussian noise to emulate insufficient detections and challenge the object tracking.



quality [time]	IMM	PAR	VMP (our method)
Gaussian	0.99 [1]	**1** [3.9]	0.98 [2.1]
GMM (**w**)	0.58 [1]	0.99 [6.37]	**1** [4.4]
GMM (**s**)	0.54 [1]	0.96 [6.2]	**1** [4.5]

5. Summary and conclusions

- We presented a novel estimation filter approach based on the framework of graphical stochastical models and variational message passing inference.
- Our approach is capable of **dynamically adjusting estimation accuracy** according to available computational resources.
- We evaluated the performance of this method in simulative as well as real-data tracking scenarios and we demonstrated **accuracy advantages over state-of-the-art approaches** in situations with non-Gaussian noise.

Bibliography

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[5] A. Geiger, P. Lenz, C. Stiller and R. Urtasun Vision meets Robotics: The KITTI Dataset. International Journal of Robotics Research, Vol. 32, pages 1229–1235, 2013