Retrospective Blind MR Image Recovery with Parameterised Motion Models

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Abstract. In this paper, we present an alternating retrospective MRI reconstruction framework based on a parameterised motion model. An image recovery algorithm promoting sparsity is used in tandem with a numeric parameter search to iteratively reconstruct a sharp image. Additionally, we introduce a multiresolution strategy to restrict the numeric complexity. This algorithm is then tested in conjunction with a simple motion model on simulated data and provides robust and fast reconstruction of sharp images from severely corrupted k-spaces.

1 Introduction

In the context of medical imaging, magnetic resonance imaging (MRI) is particularly sensitive to patient movement during data acquisition. Even though the scanning process was sped up considerably during recent years through improvements of scanner hardware and image recovery algorithms, the duration of a full scan often is prohibitively long even for cooperative patients. In addition, some involuntary movements - such as pulsatile expansions and contractions due to blood flow or intestinal peristalsis - are impractical to stop for a scan.

Autofocussing recovery algorithms used to reduce these motion artifacts require only the data measured in a conventional scan and approximates motion and the sharp image by minimizing some error metric after the scan is finished. Algorithms utilizing autofocussing do not require specialized scanner hardware nor do they influence the measurement or increase acquisition time. Generally, these approaches utilize the gradient of an error function for image updates while regularizing the motion parameters over time (e.g. in [1]). However, up to this point, blindly retroactively removing nonaffine motion remains an open challenge. As these models might not be readily differentiable, prior approaches employing gradients are not readily applicable.

We propose a general framework for such an algorithm, which allows us to fit arbitrary parameterised motion models in a blind MRI reconstruction. We employ a novel regularization scheme using path searching algorithms to find suitable time-dependent parameter sets. By iteratively approximating the image using a sparse recovery approach and numerically updating the motion model, the unknown motion is compensated effectively.

2 Methods

Let $\mathbf{x} \in \mathbb{R}^{M^2}$ be a vectorized a square image and $D_{\boldsymbol{\theta}}$ a known motion model. Generally, $D_{\boldsymbol{\theta}}$ can be understood as a linear transform matrix of size $M^2 \times M^2$ with an underlying structure compactly described by a set of parameters $\boldsymbol{\theta} \in \mathbb{R}^T$.

In MRI, trajectories through k-space are evaluated approximately at once. If we measure N trajectories described by k-space coordinate vectors $\mathbf{f}_n$ with $n \in \{0, 1, \ldots N-1\}$, we are interested in the deformation of the image $D_{\boldsymbol{\theta}}\mathbf{x}$ at N points in time and therefore affected by N distinct motion parameter sets $\boldsymbol{\theta}_n$.

The whole measurement can then be described with a k-space data vector $\mathbf{y} = [\mathbf{y}_0, \ldots, \mathbf{y}_{N-1}]^T$. Each $\mathbf{y}_n \in \mathbb{C}^{U_n}$ represents a partial measurement of the k-space acquired by the trajectory through frequency coordinates $\mathbf{f}_n$ with a number of captured coefficients $U_n \ll M^2$ defined by $\mathbf{y}_n = \mathfrak{F}_{\mathbf{f}_n} D_{\boldsymbol{\theta}_n} \mathbf{x}$. In this, $\mathfrak{F}_{\mathbf{f}_n}$ is a partial Fourier transform which can be expressed as a $\mathbb{C}^{U_n \times M^2}$ matrix.

Disregarding $D_{\boldsymbol{\theta}}$ in the reconstruction of $\mathbf{x}$ will lead to ghosting artifacts which degrade image quality and, in the worst case, will make the resulting image unfit for medical evaluation. The exact recovery of $\mathbf{x}$ requires the approximation of the underlying motion. In blind MRI image reconstruction, where no additional data is acquired, we are left with the partial measurements $\mathbf{y}_n$ and the corresponding $\mathbf{f}_n$ and tasked with the simultaneous approximation of both image and the motion through $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_0, \ldots \boldsymbol{\theta}_{N-1}\}$.

2.1 Optimization Scheme

The objective function of our reconstruction problem can - and, in comparable approaches, is - generally be expressed as

$$f_{\tilde{\boldsymbol{\Theta}}, \tilde{\mathbf{x}}} = \sum_{n=0}^{N-1} \|\mathfrak{F}_{\mathbf{f}_n} D_{\tilde{\boldsymbol{\theta}}_n} \tilde{\mathbf{x}} - \mathbf{y}_n\|_2^2 \tag{1}$$

for a fixed measurement $\mathbf{y}$. To jointly approximate image and motion parameters we split the optimization problem into an alternating algorithm in which we iteratively reduce the artifacts corrupting $\mathbf{y}$ and fit the motion model to better describe the updated image. Beginning with some arbitrary starting points $\mathbf{x}^0$ and $\boldsymbol{\Theta}^0$ we seek an updated image through

$$\mathbf{x}^{k+1} = \arg\min_{\tilde{\mathbf{x}}} f_{\boldsymbol{\Theta}^{\mathbf{k}}, \tilde{\mathbf{x}}} + \alpha\, \mathcal{R}_x(\tilde{\mathbf{x}}), \tag{2}$$

where $k \in \{0, 1, \ldots K-1\}$ is the current iteration with a maximum number of iterations K. Since the problem is ill-posed, $\mathcal{R}_x$ is a regularization term for the image with Lagrange multiplier α. We then seek an updated motion parameter set through

$$\boldsymbol{\Theta}^{k+1} = \arg\min_{\tilde{\boldsymbol{\Theta}}} f_{\tilde{\boldsymbol{\Theta}}, \mathbf{x}^{k+1}} \text{ s.t. } \tilde{\boldsymbol{\Theta}} \in \Psi \tag{3}$$

which fits the parameter set to the updated image. Since acquisition time in MRI is generally short, we expect that the motion state does not change too much between consecutive time steps. To quantify this change, we define a distance function $\phi(\cdot,\cdot)$ which is used as a difference measure of parameterised motion states. Using this, we define a set of admissible motion parameter sets

$$\Psi = \{\boldsymbol{\Theta} \mid \phi(\boldsymbol{\theta}_n, \boldsymbol{\theta}_{n-1}) < \tau \ \forall\, n \in \{1, \ldots, N-1\}\} \tag{4}$$

with a maximal allowed distance between time steps τ.

We will now describe the two update steps and their motivation in greater detail while providing some implementation details.

2.2 Image reconstruction

The optimization probem in Eq. (2) is regularly found in literature and research concerning sparse data recovery. In the latter, the regularization term is chosen so that structure of the underlying data is exploited and a solution is chosen which is sparse in a transform basis.

Using this approach, we chose $\mathcal{R}_x(\mathbf{x}) = \|\mathbf{W}\mathbf{x}\|_1$, where $\mathbf{W} \in \mathbb{R}^{M^2 \mathrm{x} M^2}$ is an invertible wavelet transform matrix. It is well known from literature that natural images can be represented sparsely in the wavelet domain. Since the ghosting artifacts afflicting the image reduce its sparsity, we can expect to improve image quality by this approach.

Thus, Eq. (2) is an unconstrained convex optimization problem which can be solved in numerous ways. Because of the size of the involved system matrices, algorithms requiring system matrix inversion are not feasible for larger images. We instead use the (scaled) alternating direction method of multipliers (ADMM)[2] to find a solution iteratively.

Using the ADMM framework, the problem can be decomposed into an alternating algorithm consisting of the three sub-steps

$$\mathbf{u}^{j+1} = \arg\min_{\tilde{\mathbf{u}}} \sum_{n=0}^{N-1} ||\mathfrak{F}_{\mathbf{f}_n} D_{\tilde{\boldsymbol{\theta}}_n} \tilde{\mathbf{u}} - \mathbf{y}_n||_2^2 + \frac{\lambda}{2}\|\tilde{\mathbf{u}} - \left(\mathbf{v}^j + \mathbf{w}^j\right)\|_2^2 \tag{5}$$

$$\mathbf{v}^{j+1} = \mathcal{T}_{\alpha/\lambda}\left(\mathbf{W}\left(\mathbf{u}^{j+1} + \mathbf{w}^j\right)\right), \quad \mathbf{w}^{j+1} = \mathbf{w}^j + \mathbf{u}^{j+1} - \mathbf{W}^{-1}\mathbf{v}^{j+1} \tag{6}$$

starting from some arbitrary $\mathbf{u}^0$, $\mathbf{w}^0$ and $\mathbf{v}^0$ with j denoting the iteration and λ being the penalty parameter of the augmented Lagrangian. With a soft-thresholding function $\mathcal{T}_z(\cdot)$ with threshold z, the updates for $\mathbf{v}^{j+1}$ and $\mathbf{w}^{j+1}$ are trivial. The update of $\mathbf{u}^{j+1}$ can be solved using convex optimization - we used a conjugate gradient descent to find an update.

2.3 Motion Update

The optimization of $\boldsymbol{\Theta}$ is unfortunately not as straight-forward for arbitrary motion models $D_{\boldsymbol{\theta}}$. Since we seek a general solution independent of the underlying motion model, we cannot optimize using convex optimization.

The objective function Eq. (1) however can be split to a number of N smaller problems by splitting the sum. For a fixed measurement and image approximation we end up with

$$\hat{f}^n_{\boldsymbol{\theta}} = ||\mathfrak{F}^H_{\mathbf{f_n}} \mathbf{y_n} - D_{\boldsymbol{\theta}} \mathbf{x}^{k+1}||_2^2. \tag{7}$$

By defining a search space for the parameter set, we can then numerically evaluate every *partial* objective function $\hat{f}^n_{\boldsymbol{\theta}}$ on possible values for $\boldsymbol{\theta}_n$ and look for the set which minimizes Eq. (1) while being in Ψ.

To sample the partial objective function with respect to $\boldsymbol{\theta}$, we create a parameter lattice $\Omega \subset \mathbb{R}^T$. We approximate a maximum absolute value G_t and a step size γ_t for each value in the parameter representation $\boldsymbol{\theta}_t$, so that we end up with

$$\Omega = \left\{ \sum_{t=0}^{T-1} a_t \boldsymbol{\theta}_t \mid a_t \in \{-G_t, -G_t + \gamma_t, \ldots, G_t\} \right\}. \tag{8}$$

The parameters G_t and γ_t are crucial for the algorithm, as they define the maximum value for every parameter as well as its resolution.

Using a linear index $v \in \{0, 1, \ldots, V-1\}$ with $V = \prod_{t=0}^{T} \left\lceil \frac{2G_t}{\gamma_t} \right\rceil$ we then calculate the error term for every motion state $\boldsymbol{\omega}_v$ on the lattice and collect them into a matrix $\mathbf{Q} \in \mathbb{R}^{N \times V}$ with entries $q_{n,v} = \hat{f}^n_{\omega_v}$.

The update of $\mathbf{\Theta}^k$ then reduces to a path search problem through $\mathbf{Q}$. We use a modified variant of the Viterbi algorithm [3], but other path search algorithms might be applicable as well.

For the path search algorithm, we initialize a path storage $p_{0,v} = v$, state transition sets $\beta_v = \{\boldsymbol{\omega}_h \mid \phi(\boldsymbol{\omega}_v, \boldsymbol{\omega}_h) \leq \tau\}$ and a path error matrix $\mathbf{E} \in \mathbb{R}^{N \times V}$ with entries $e_{0,v} = q_{0,v}$. We then update paths and error matrix by

$$\tilde{p}_v = \underset{c \,\in\, \beta_v}{\arg\min} \left[e_{n-1,c} \right], \quad e_{n,v} = e_{n-1,\tilde{p}_v} + q_{n,v}, \quad p_{n,v} = [p_{n-1,v},\, \tilde{p}_v] \tag{9}$$

while iterating through $n = \{1, \ldots, N-1\}$. The 'optimal' path $p_{j,N-1}$ corresponding to the smallest $e_{N-1,v}$ then defines the updated motion parameter set by $\mathbf{\Theta}^{k+1} = \{\boldsymbol{\omega}_{p_{j,0}}, \ldots, \boldsymbol{\omega}_{p_{j,N-1}}\}$. This formulation allows us to find the optimal path in $\mathcal{O}(VN)$, once $\mathbf{Q}$ is calculated.

Due to the nature of $\mathbf{Q}$, accurately approximating a large number of parameters in a big region would be infeasible using this approach. However, if the partial objective functions are 'smooth enough' for a step size γ_t we can utilize a different approach. Refining the grid between iterations by progressively lowering G_t and γ_t and centring the search space on $\mathbf{\Theta}^{k-1}$ - that is, evaluating $q_{n,v} = \hat{f}^n_{(\boldsymbol{\omega}_v - \boldsymbol{\theta}^{k-1}_n)}$ - yields a multiresolution approach. This approach will fail if the partial objective function changes rapidly for a step size γ_t, which depends on the motion model and the underlying image.

Regardless whether the grid is refined or not, the update formulation comes with a slight caveat: The resulting motion set is ambiguous in that every $\mathbf{\Theta}$ which fulfills $\mathbf{y}_n = \mathfrak{F}_{\mathbf{f}_n} D_{\boldsymbol{\theta}_n} D_{\boldsymbol{\xi}} \mathbf{x}$ is a possible solution. In essence, we cannot directly influence the 'stating state' of $\mathbf{x}$, which might still be warped with a constant bias motion $D_{\boldsymbol{\xi}}$. Although this might be unproblematic in practical situations, we circumvent this problem after the final iteration of the algorithm. Modifiying $\boldsymbol{\Theta}^{K-1}$ with a bias term $\boldsymbol{\kappa} \in \mathbb{R}^T$ yields a final set $\hat{\boldsymbol{\Theta}} = \{\boldsymbol{\theta}_0^{K-1} - \boldsymbol{\kappa}, \ldots, \boldsymbol{\theta}_{N-1}^{K-1} - \kappa\}$ so that $D_{\hat{\boldsymbol{\theta}}_0}\mathbf{I} = \mathbf{I}$ and the image at the $n = 0$ is recovered.

2.4 Test Setup

Up to this point, we have outlined a general reconstruction framework for arbitrary motion. The model driving the reconstruction is part of ongoing research. However, we still evaluate the performance using a simplistic model to provide preliminary results.

We used PROPELLER [4] (Periodically Rotated Overlapping ParallEL Lines with Enhanced Reconstruction) as a k-space acquisition scheme with 50 blades. Translational movements of the underlying image between blade acquisitions were used to corrupt the measured k-space. This motion model results in a very small set of parameters ($T = 2$) and an intuitive parameter distance function $\phi(\boldsymbol{\theta}_a, \boldsymbol{\theta}_b) = \|\boldsymbol{\theta}_a - \boldsymbol{\theta}_b\|_2$. As a base image, we used a simulated tomography of the human brain [5] with a size of 150×150 pixels.

Approximately smooth random motion curves were used with a varying maximum amplitude of $\boldsymbol{\theta}_{\max}$.

Preliminary numerical evaluations show the partial objective functions to be almost convex in a region around the true minimum. Therefore, we utilize multiresolution and refine the lattice between iterations. Starting with a maximum allowed motion of $G_t = 30$ pixels and step size $\gamma_t = 3$ for both parameters, both were halved every three iterations. The maximum number of iterations was fixed to $K = 18$. The starting step size was found to be inconsequential and fixed to $\gamma_t = 3$. The maximum parameter distance τ was set to $2/10 \cdot G_t$.

Parameters concerning image recovery were chosen empirically - we used a separable wavelet transform with a Daubechies mother wavelet, a shrinking parameter $\alpha = 1/2$ and an ADMM penalty parameter of $\lambda = 1$.

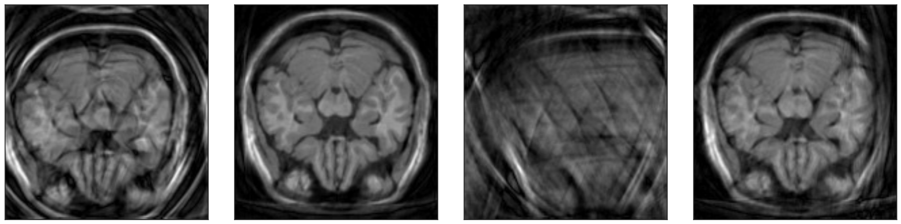
For error measurements we used an l_2-norm between base image and reconstruction to quantify the recovery error as well as the structural image similarity measure (SSIM) and total variation (TV) for artefact quantisation.

3 Results

Fig. 1 shows the simulated data prior to the algorithm and after its final iteration. Sharp images are recovered even from harshly corrupted k-spaces, while the multiresolution iteration allows for the improvement of images with $\boldsymbol{\theta}_{max} > G_t$. We expect the image to further improve with more iterations of the algorithm.

Fig. 2 shows the error measurements at the last iteration of the algorithm for different $\boldsymbol{\theta}_{\max}$ as well as different starting values for γ_t. As expected, the l_2-error is robustly decreased for all $\boldsymbol{\theta}_{\max}$, yielding near perfect results for small movements[1] while SSIM is increased.

[1] Note that PROPELLER oversamples central regions of k-space while undersampling edge regions, which in itself yields an l_2-error even for unmoving subjects.



(a) $\boldsymbol{\theta}_{\max} = 10$ (b) Reconst. (c) $\boldsymbol{\theta}_{\max} = 40$ (d) Reconst.

Fig. 1. Corrupted image and reconstruction for two maximal motion amplitudes.

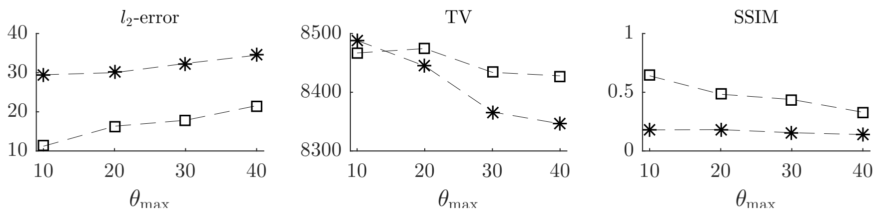


Fig. 2. Error values for images before (cross) and after (square) reconstruction

Recovery of a sharp image using this approach takes about 5 minutes.

4 Discussion

In this work, we present an algorithm with which the parameter set of an a-priori defined motion model can be approximated using partially sampled k-spaces. This algorithm can be used to find a robust approximation of both sharp image and corrupting motion simultaneously. Although it can be computationally intensive, we propose a scheme to find a good solution with a reduced number of numeric evaluations using multiresolution. The development of suitable low-parametric motion models is subject of ongoing research.

Acknowledgement This work has been supported by the German Research Foundation under Grant No. ME 1170/11-1.

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