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Graphical stochastic models for tracking applications with variational message passing inference

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Abstract— **In this paper we present a novel, highly-adoptable, state-estimation filter based on the framework of graphical stochastical models and variational message passing inference. We evaluate our method on both real and simulated data for tracking applications.**
Our experimental results show that the proposed approach offers qualitative and computational advantages over established filter methods in practical situations, where the noise within a process is not simply a Gaussian noise, but rather described by a more complex distribution.

Keywords— **Object tracking, Image-based object tracking, Non-Gaussian noise, Variational inference, Particle filter, KITTI**

I. Introduction

In many practical applications, such as tracking applications, we need to deal with observations from a system whose internal mechanics are too complicated to be modeled directly. The method of choice in such cases is to use a statistical model, that allows us to proceed by projecting our lack of knowledge on an uncertinty term within an established mathematical formalism. Within this framework, ordered observations (in particular time-ordered) are related to stochastic processes. These are thus models for systems whose unknown internal states undergo a process thus changing from observation to observation. The estimation of the internal state, therefore, is a very challenging task. In general this estimation requires the application of a state-estimation filter. These filter methods allow the usage of prior system knowledge, as well as complementary information in order to achieve the best possible state estimation.
Extensive research has been conducted to design models for processes with additive and Gaussian distributed measurement and process noise. Probably the most popular representatives of this filter class are the Kalman filter [15] and its derivatives, such as extended Kalman filter [9] or unscentend Kalman filter [14].
Nevertheless in many situations, the requirements for the usage of one of these filters are not fulfilled. While the assumption of additivity in the case of noise is often still feasible, the assumption of zero-mean Gaussian distribution is not. These situations require a more general and flexible filter approach. An approach such as the particle filter [21] or an interacting multi model filter (IMM) ([6],[8],[17],[19]).
However both methods have also significant weaknesses and limitations. For example, a particle filter often struggles with side effects of the sampling based approximation of the a posterori density. Effects such as high computational effort or informative depletion ([12],[22]). An IMM-filter, however, is limited by the boundary conditions of the underlying filter ensemble.
In this contribution, we introduce a novel filter approach to the problem of state estimation in the context of hidden Markov processes. A filter approach which is developed within the **framework of graphical stochastical models** as well as **variational message passing**. In Section IV we will argue that this proposed method provides notable qualitative and numerical advantages over state-of-the-art methods in multiple practical situations.

II. VMP Inference in chosen probalistic graphical models

The research around graphical stochastical models is very active and a constant source of many publications ([1], [3],[16]).
This activity can be explained by fact, that the concept of graphical stochastical models, including the existing modern inference methods, represents a powerful and very generic mathematical tool for modeling the behavior of coupled stochastic processes.
An exemplary stochastic process, whose behavior is accurately emulated in the form of a generative graphical stochastical model, this is the one behind a Kalman filter (see Fig. 1). The nodes in this graphical model depict random variables.

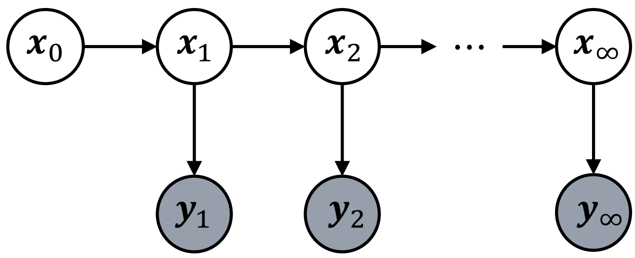

Fig. 1. Generative graphical model of a hidden Markov process.

They represent the time evolving latent system states (white nodes) as well as corresponding measurable observables (dark nodes).
The directed edges reflect the time transitions between the latent system states and the causal relationships between

system states and their corresponding observables. Thereby they indirectly represent the existing conditional probabilities between those nodes.

In consequence of its cycle-free structure, this stochastic process model is classifiable as a linear Bayesian network [5], whose corresponding joint distribution can be formulated, based on the existing likelihoods, as follows:

$$P\left(\boldsymbol{H}_{0:N}, \boldsymbol{Y}_{1:N}\right) = P\left(\boldsymbol{H}_0\right) \cdot \prod_{i=1}^{N} P\left(\boldsymbol{H}_i | \boldsymbol{H}_{i-1}\right) \cdot P\left(\boldsymbol{Y}_i | \boldsymbol{H}_i\right) \ . \tag{1}$$

This definition of the joint distribution is the starting point of all inference. And in this sense it is the origin for the calculation of estimation filter essential marginal $P\left(\boldsymbol{H}_k | \boldsymbol{Y}_{1:k}\right)$.

If the dynamic and measurement behaviour of a process can be described accurately by linear models and if all noise within the process behaves according to zero-mean Gaussian noise, the solution for this calculation can be derived in a closed-analytical form, as evidenced by the Kalman equations [15]. But if the characterisation of the process differs from these strict conditions, the analytical calculation of the desired marginal $P\left(\boldsymbol{H}_k | \boldsymbol{Y}_{1:k}\right)$, in general, fails.

In order to solve this problem, we propose a deterministic-working numerical inference approach called **variational message passing (VMP)**.

The principle of this inference approach was proposed by Winn et al. ([23],[7]). The motivation for this approach was the development of a deterministic method, which can be used to optimally approximate the characteristic likelihood function $P\left(\boldsymbol{H}|\boldsymbol{V}\right)$ of a process, based on a generated sampling set $\boldsymbol{V}$, with $\boldsymbol{V} = \{\boldsymbol{V}_1, \ldots, \boldsymbol{V}_N\}$, by a tractable likelihood function $Q\left(\boldsymbol{H}\right)$, according to:

$$Q^*\left(\boldsymbol{H}\right) \approx P\left(\boldsymbol{H}|\boldsymbol{V}\right). \tag{2}$$

Winn et al. developed their approach based on the variational inference idea, which was introduced by Attias ([2]), Ghahramani and Beal ([11]) and Bishop ([7]). Essentially, it uses the variational equation

$$\begin{aligned}\ln\left(P\left(\boldsymbol{V}\right)\right) &= \sum_{\boldsymbol{H}} \ln\left(\frac{P\left(\boldsymbol{H},\boldsymbol{V}\right)}{Q\left(\boldsymbol{H}\right)}\right) \cdot Q\left(\boldsymbol{H}\right) \\ &\quad - \sum_{\boldsymbol{H}} \ln\left(\frac{P\left(\boldsymbol{H}|\boldsymbol{V}\right)}{Q\left(\boldsymbol{H}\right)}\right) \cdot Q\left(\boldsymbol{H}\right) \\ &= \mathcal{L}\left(Q\right) - KL\left(Q||P\right), \end{aligned} \tag{3}$$

to indirectly identify an optimal approximation of the desired likelihood $P\left(\boldsymbol{H}|\boldsymbol{V}\right)$ in form of a tractable function $Q\left(\boldsymbol{H}\right)$, based on the well-known definition of the joint distribution, according to

$$P\left(\boldsymbol{H},\boldsymbol{V}\right) = P\left(\boldsymbol{Z}\right) = \prod P\left(\boldsymbol{Z}_i, Pa\left(\boldsymbol{Z}_i\right)\right). \tag{4}$$

This idea allows us to solve the main problem, the minimum problem of Kullback-Leibler divergence $KL\left(P\left(\boldsymbol{H}|\boldsymbol{V}\right)||Q\left(\boldsymbol{H}\right)\right)$, which leads to the desired approximation $Q\left(\boldsymbol{H}\right)$, by solving the complementary maximization problem $\mathcal{L}\left(Q\right)$, the so-called lower-bound problem:

$$\underset{Q}{\operatorname{argmin}}\left(KL\left(Q||P\right)\right) = \underset{Q}{\operatorname{argmax}}\left(\mathcal{L}\left(Q\right)\right) \tag{5}$$

For developability of an inference approach, based this constructed complementary lower-bound problem, Winn et al. [23] introduced some additional requirements:

Acyclic Graph: The stochastic process has to be accurately modeled by an acyclic graphical Bayesian network.

Factorisation: For the mathematical handling, the solution space of $Q\left(\boldsymbol{H}\right)$ is restricted. Only those solutions that meet the condition

$$Q\left(\boldsymbol{H}\right) = \prod_i Q_i\left(\boldsymbol{H}_i\right) \tag{6}$$

are considered. This so-called mean-field approximation [5] decouples the influence of a random variable $\boldsymbol{H}_i$ on $Q\left(\boldsymbol{H}\right)$ by introducing variable-individual partial functions $Q_i\left(\boldsymbol{H}_i\right)$.

Exponential Family: Each likelihood $P\left(\boldsymbol{X}_i|Pa\left(\boldsymbol{X}_i\right)\right)$, with $\boldsymbol{X} = \left(\boldsymbol{H}, \boldsymbol{V}\right)$ being a component of the stochastic process (Eq. 4) must be characterisable as a function of the exponential family [18]. So their mathematical forms follows the structure:

$$\begin{aligned}\ln\left(P\left(\boldsymbol{X}_i|Pa\left(\boldsymbol{X}_i\right)\right)\right) &= \boldsymbol{\phi}_{X_i}\left(Pa\left(\boldsymbol{X}_i\right)\right)^T \cdot \boldsymbol{u}_{X_i}\left(\boldsymbol{X}_i\right) \\ &\quad + f_{X_i}\left(\boldsymbol{X}_i\right) + g_{X_i}\left(Pa\left(\boldsymbol{X}_i\right)\right) \ , \end{aligned} \tag{7}$$

where $\boldsymbol{\phi}_{X_i}\left(\cdots\right)$ is the so-called **natural statistic vector** and $\boldsymbol{u}_{X_i}\left(\cdots\right)$ is the **natural parameter vector**. This requirement is fulfilled by a large selection of unimodal and multimodal distributions. Such as exponential, Wishart, Dirichlet, Discrete or, with a little trick, Gaussian mixture distributions.

Based on these boundary conditions the proposed VMP algorithm of Winn et. al recognises the optimal approximation $Q^*\left(\boldsymbol{H}\right)$ by identifying the optimal partial functions $Q_i^*\left(\boldsymbol{H}_i\right)$. For this purpose the algorithm uses a specific iterative message-passing scheme, which constantly exchanges and allocates information between the nodes. This scheme finally identifies the optimal partial functions $Q_i^*\left(\boldsymbol{H}_i\right)$ that maximise the lower-bound $\mathcal{L}\left(Q\right)$ and minimise the Kullback-Leibler divergence $KL\left(Q||P\right)$.

III. Iterative formulation of VMP based estimation filter

If a stochastic process can be accurately represented by a linear-chain graphical model as described in Fig. 1 and if it is also compatible to the previous mentioned restrictions (Sec. II), then the VMP inference method allows us to create a state-estimation filter for this exact process.

By directly applying the VMP inference to such a stochastic process, the result will be a tractable approximation of the likelihood $P\left(\boldsymbol{H}|\boldsymbol{V}\right)$ in the form of

$$P\left(\boldsymbol{H}|\boldsymbol{V}\right) \approx Q^*\left(\boldsymbol{H}\right) = \prod_i Q_i^*\left(\boldsymbol{H}_i\right) \ . \tag{8}$$

Even if this factorised definition easily allows to infer arbitrary likelihoods, a typical state estimation filter is just interested in a specific likelihood: The marginalization $P(\boldsymbol{H}_k|\boldsymbol{V}_{1:k})$, which represents the distribution of last system state $\boldsymbol{H}_k$ based on all previous measurements. For practical reasons a state-estimation filter simplifies that likelihood to its corresponding stochastic moments. In this sense, an estimation of the last system state $\boldsymbol{H}_k$ is defined according to

$$\tilde{\boldsymbol{H}}_k = \langle P(\boldsymbol{H}_k|\boldsymbol{V}_{1:k})\rangle = \langle Q_k(\boldsymbol{H}_k)\rangle \tag{9}$$

as the most probable realisation of the latent random variable $\boldsymbol{H}_k$, inferred from the corresponding likelihood $Q_k^*(\boldsymbol{H}_k)$.
But as Eq. 4 indirectly implies, a naive application of the VMP inference on a time-infinitely progressing stochastic process, like previously discussed, would inevitably cause critical issues, because an estimation filter based on the growing graphical model of the process would simply numerically diverge over time.
Since this circumstance complicates the practical usage of such a estimation filter, we propose the following measures:

Motivated by an assumed Markovian character of the considered process, we introduce a **sliding window concept**, which, starting from the current time k, limits, in the perspective of VMP inference approach, the back lying history of the process. In this regard, state estimations from earlier time steps (which have left the sliding window) are declared as constants or observables, based on their last known estimation $\boldsymbol{H}_i$. So, for the estimation of the last system state $\boldsymbol{H}_k$ these nodes outside of the sliding window are now insignificant.
This set of rules is now graphically reflected in the schematic diagram in Fig. 2.
We point out that in order to receive a numerical efficient implemention, we recommend to choose the sliding window size, with regard to the characteristics of the process and the estimation accuracy, as small as possible.
Finally, the algorithm of the proposed estimation filter summarises to the pseudocode shown in Alg.1. We like to mention that the form of the presented algorithm is completely generic. Consequently, the proposed approach is suitable for all those time-driven stochastic processes, whose stochastic behavior is accurately emulated by a graphical model shown in Fig. 2.
In this respect, the proposed filter concept is compatible and accessible to a large range of stochastic processes and can therefore be used in a variety of applications.

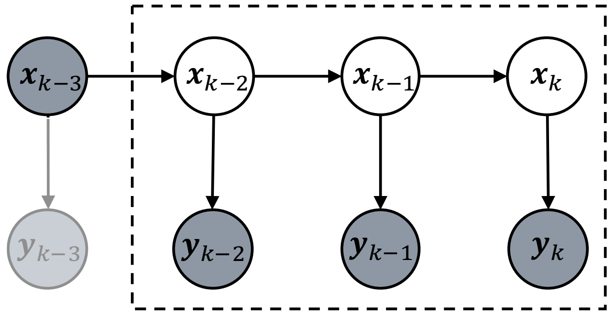

Fig. 2. Proposed sliding window concept (e.g. $size = 3$).

Algorithm 1 VMP-Based Estimation Filter

% **Initialisation Of Basic Parameter**
$\boldsymbol{x}_0 \leftarrow \boldsymbol{x}_{Init}$ ▷ $\boldsymbol{x}_{Init}$: Initial State
$\boldsymbol{P}_0(\boldsymbol{x}) \leftarrow \boldsymbol{P}_{Init}(\boldsymbol{x})$ ▷ $\boldsymbol{P}_{Init}(\boldsymbol{x})$: Initial State Distribution
$\boldsymbol{sP}_0 \leftarrow \boldsymbol{sP}_{Init}$ ▷ $\boldsymbol{sP}_{Init}$: Filter Structure Parameter

% **Initialisation Of Graphical Filter Structure**
$graph_0^* \leftarrow f_{ConfigFilterGraph}(\boldsymbol{x}_0, \boldsymbol{P}_0, \boldsymbol{sP}_0)$

% **Estimation**
while 1 **do**

% **Modify Graph Configuration**
$graph_k \leftarrow f_{ModifyFilterGraph}\left(graph_{k-1}^*, \boldsymbol{y}_k, \boldsymbol{u}_k\right)$

% **Execute VMP Inference**
$graph_k^* \leftarrow f_{Inference}(graph_k)$

% **Extraction Of Estimation**
$\boldsymbol{x}_k \leftarrow f_{ExtractEstimation}(graph_k^*)$

end while

IV. Results and Discussions

In the following section we will analyse the performance of the proposed filter approach. For this purpose, we will evaluate the filter in the context of simulated and real object tracking scenarios that are each quite challenging for conventional filter approaches. In order to validate the results of the proposed method, we compare it with various state-of-the-art methods.

A. Simulated Scenearios: Tracking of vehicle

The considered test scenarios are different-configured cornerings of a vehicle in a two-dimensional space. The simulated movements of the tracked vehicle will be fully accurately described by a linear CWPA-model [24]. Hence, we have an accurate process model. The specific pathway of the trajectories will be determined over temporarily active accelerations in horizontal and vertical direction.
Moreover, we assume that the position of the moving vehicle is measured by an external sensor. Occurring measurement errors within the sensor are assumed to be distributed according to a Gaussian mixture model (GMM). Overall, the characteristics of the test data-generating process, summarises as follows:

$$\begin{aligned}\boldsymbol{W}_k &\sim \sum_i a_i \cdot N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)\ , \\ \boldsymbol{H}_{k+1} &= \boldsymbol{A}_k \cdot \boldsymbol{H}_k + \boldsymbol{B}_k \cdot \boldsymbol{U}_k\ , \\ \boldsymbol{V}_k &= \boldsymbol{C}_k \cdot \boldsymbol{H}_k + \boldsymbol{W}_k\ .\end{aligned} \tag{10}$$

To be as close as possible to real conditions in tracking applications, all the subsequently discussed estimation filters are implemented as unknown-input systems. Therefore,

only the sensor-measured position-signals affect the state-estimation.
In advance, it should be noted that we created all of the following results by multiple reruns of the corresponding experiments. Stochastic fluctuations within the estimation results are therefore opposed as best as possible.
In order to respond to the stochastic characteristics of the test data generating process, our proposed filter will be based on the following process emulating graphical model Fig. **??**.

Our considered selection of state-of-the-art methods is orientated towards the challenging non-Gaussian measurement noise within the simulated process. Therefore, we will consider to two different state-of-the-art methods.

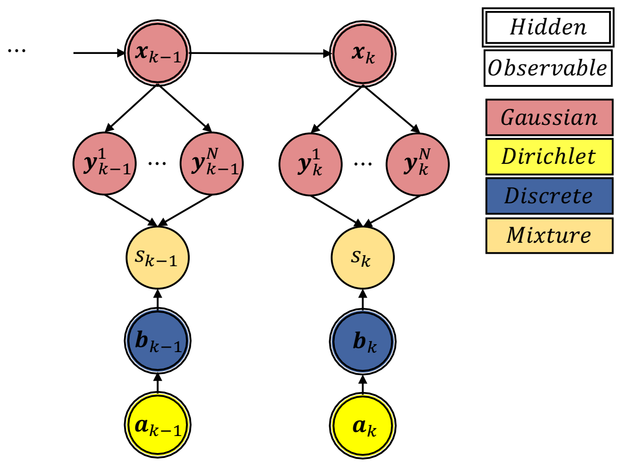

Fig. 3. Graphical model of the considered test process.

The first method is a particle filter [21]. Its configuration has been chosen as follows: Both the measurement and process model, as well as the measurement and process noise characteristics are the direct correspondence of the configuration of simulated process (see Eq. 10).
Its initial MCMC-sampled particle set [4] corresponds to a normal distribution around the initial filter state. It covers $N = 200$ particles. The resampling is happing in accordance to a residual strategy [13]. Thus, the number of particles will always be constant. The subsequent shift of all resampled particles is done by a Gaussian kernel function.
The second state-of-the-art method is an IMM-filter ([6],[8]). To optimally respond to both aspects in the considered cornering scenarios, a variant movement dynamic of the vehicle along the trajectory and a desired efficient filter implementation, we choose filter ensemble with four different designed Kalman filters.
In detail, the ensemble includes two CWPA-model-based filters for sinuous trajectory sections, as well as two CWNA-model-based for straight sections. The measurement and the process model of CWPA-model-based filters, comparable to the configuration of the particle filter above, directly correspond to those of the simulated process (Eq. 10). Therefore, the CWPA-model-based filters do not consider any kind of process noise. This changes in the context of CWNA-model-based filters. As consequence of inaccurate emulated motion dynamics, these filters have to consider lightweight process noise.
The diversity between our identical CWNA-[24] or CWPA-model-based filters results from different chosen measurement noise configurations (Eq. 10). Within the limits of the Kalman filter, this diversity is used to optimally adapt to the non-Gaussian distributed measurement noise. Since the simulated trajectories are characterised by varying dynamics, the initial model weights and transition matrix of the IMM-filter are parameterised neutrally. Consequently, no particular filter of the ensemble is emphasised by the initialisation.

The achieved results of the filter comparison for three different measurement noise configurations are now presented in Fig. 4, 5 and 6 in form of the estimated trajectories, as well as in table 1 in form of the corresponding computational effort analysis.
These three considered measurement noise configurations correspond to Gaussian (g), weak Gaussian mixture model (w) and strong Gaussian mixture model (s) distributed noise. In advance to the subsequent discussion of the results, we point out that all trajectory-estimation-containing figures apply to the following scheme: Calculated trajectory-estimations are drawn in color, while the ground truth trajectory is drawn in dotted-black.

If we first analyze the achieved results of the considered filters in the case of Gaussian (g) distributed measurement noise, it can be noted that the estimations of the various methods are not only almost identical, but also very accurate. All three filter concepts are directly benefiting from the shape of the measurement noise.

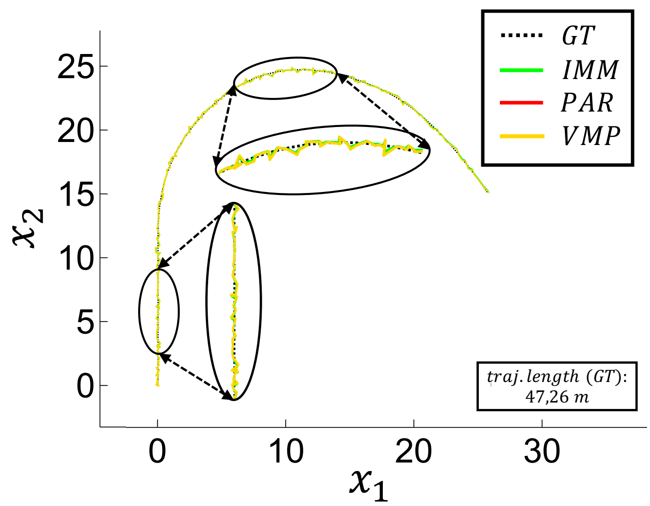

Fig. 4. Estimated trajectories in case of Gaussian distributed measurement noise.

But this, however, changes if the shape of measurement noise

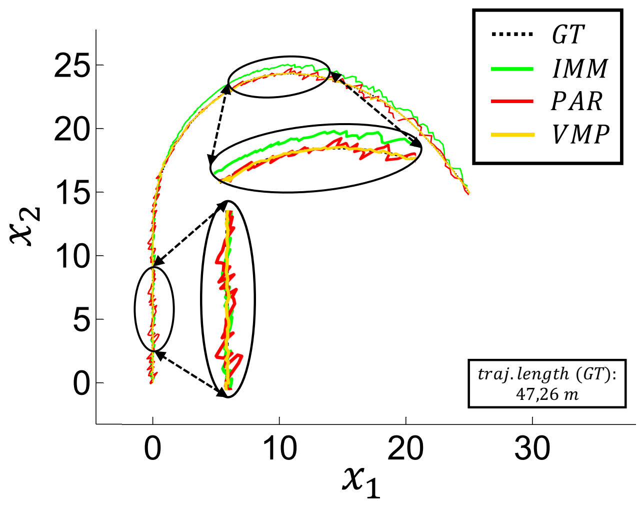

Fig. 5. Estimated trajectories in case of weak Gaussian mixture model distributed measurement noise.

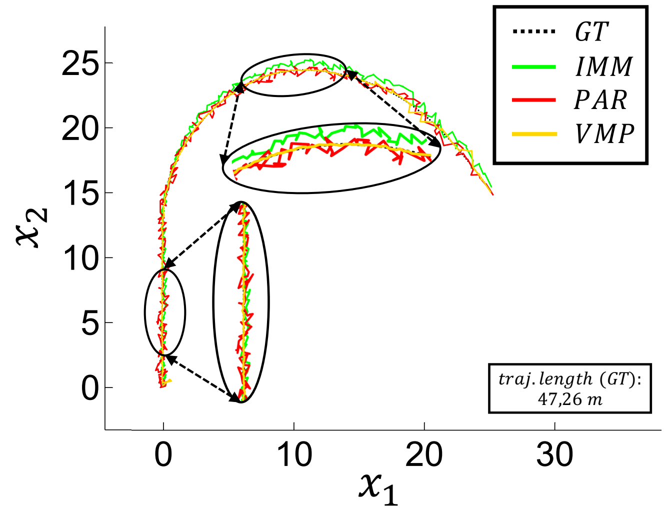

Fig. 6. Estimated trajectories in case of strong Gaussian mixture model distributed measurement noise.

is modified. This is easily noticeable in the case of weak Gaussian mixture model (w) distributed measurement noise (see Fig. 5).

Here, in this situation the proposed VMP-based filter differs qualitatively from the other two estimation methods. The new shape of the measurement noise now formally violates the application requirements of the Kalman filter-based IMM-filter, which ultimately causes an inaccurate IMM-filter estimation. On the contrary, the VMP-based filter estimation is not much effected. With a few exceptions, the filter identifies that GMM-component of the simulated measurement noise, which is responsible for the noise impact to time step k and estimates the system state $\boldsymbol{H}_k$, unlike the IMM-filter, very accurately. Something similiar is also achieved by the particle filter. In consequence of its design, the filter also manages to react appropriately to the changed shape of the noise. The calculated estimations are fairly accurate in comparison to ground truth pathway. Although, if compared with the VMP-based filter, the estimated trajectory is noticeably noisier. However, more striking than the qualitative difference between these two methods is the difference between their average computational effort for calculating these estimations (see Table 1). As the table reflects, the numerical effort of the particle filter exceeds the numerical effort of the VMP-based filter several times. In that context we point out that here the IMM-filter requires the least computational effort.

This described constellation of the computational effort between the three compared filter methods formally maintains even after increasing the measurement noise distribution towards a more extensive Gaussian mixture model (s) (see Fig. 6). Qualitatively, however, the differences between the trajectory estimations of the diverse filter methods are now more pronounced than before. Even with the more complex measurement noise distribution, the proposed VMP-based filter continues to produce a relatively accurate trajectory estimation. The two other methods are failing in doing so. The trajectory estimation of the particle filter formally follows the ground truth, but due to its lack of compensating the strong measurement (s), it is very noisy. The same applies for the estimated trajectory of the IMM-filter. Moreover, as a result of the partly inaccurate dynamic assumption (influences of the CWNA-model-based ensemble part) the IMM-estimated trajectory also has, similar to the previous situation of weaker measurement noise (w), a larger bias to the ground truth trajectory.

Finally, by reflecting all the previously discussed results, we retain that the proposed filter approach stands out of the state-of-the-art estimation filter methods.

The IMM-filter is qualitatively outperformed by the proposed filter approach in situations with non-Gaussian distributed measurement noise. Similar is the situation in the case of the particle filter. But here the filter is not only outperformed qualitatively, but also computationally.

Table 1. Average computational effort per iteration relative to the fastest method (smaller is better) and in *brackets* relative accumulated L2-estimation-error between estimation and ground truth relative to the most accurate method (smaller is better).

	Gaussian-noise (**g**)	GMM-noise (**w**)	GMM-noise (**s**)
IMM	**1** [1.1]	**1** [4.4]	**1** [4.1]
PAR	202.4 [1.1]	245.8 [2.0]	275.1 [1.5]
VMP	23.7 [**1**]	38.2 [**1**]	54.3 [**1**]

B. *Real Scenarios: Tracking of detected objects*

In addition to the considered simulative environment, we evaluate the discussed filter methods in the context of real-data image-based object-tracking. For this purpose we exploit the KITTI-framework [10]. Within this framework, we use the estimation filter for improving the positioning of classifier-detected objects (e.g. cars) [20] in 2D image space by including track-information and model-knowledge of the object movement. In order to extensively survey the performance of the proposed approach, we consider three different constellations within this application context: In a first constellation we improve the positioning of the classifier-detected object-measurements, which are assumed as Gaussian distributed. In the second and third constellation, we first artificially superimpose the original classifier-detected object measurements by adding different magnitudes of GMM-noise to simulate an insufficiently-trained classifier and then try to improve the positioning of these noisy object-detections with help of the filters.

For validation reasons, we compare the generated object-bounding-boxes of the different filter methods with manually labeled object-bounding-boxes of the KITTI-framework based on a catalog of 2945 frames with 78 different object-tracks. The results of this analysis, along with corresponding computational efforts, are presented in Table 2. The results show that all the filters provide nearly the same estimation performance, if the observable data is accurately modeled as a Gaussian-distributed signal. But this changes when the original classifier-detections are superimposed by additional GMM-noise. Here, the particle filter and the VMP-based filter are able to excel significantly from the IMM-filter. Whereby we outline that the VMP-based filter achieves the same performance as a particle filter, but with less computational effort. So our proposed VMP-based filter seems to be a capable method for this kind of real-data constellation.

Table 2. Average computational effort per iteration relative to the fastest method (smaller is better) and in *brackets* quality measure relative to the best method (higher is better).

	none active noise	GMM-noise ($\mathbf{w}$)	GMM-noise ($\mathbf{s}$)
IMM	**1** [0.99]	**1** [0.58]	**1** [0.54]
PAR	3.9 [**1**]	6.37 [0.99]	6.2 [0.96]
VMP	2.1 [0.98]	4.4 [**1**]	4.5 [**1**]

V. Conclusions and Further Work

In this paper we presented a novel estimation filter approach based on the framework of graphical stochastical models and variational message passing (VMP) inference. We evaluated the performance of this method in the context of various trajectory estimations in a simulative and real-data environment.

We showed that the presented filter approach offers accuracy advantages over an IMM- or a particle filter in situations where the noise within the process is not simply Gaussian distributed, but rather described by a more complex distribution.

We also indirectly presented that the lower-level VMP inference allows the proposed filter approach to dynamically adjust its estimation accuracy according to available computational resources. This is a very valuable feature in real-time capable filter implementations.

In future work, we plan to expand the generality of the approach. Therefore, our development will be focusing on two central aspects. The first is the obvious transfer of the approach to processes with nonlinear process and measurement models as well as non-Gaussian process noise. The second is the adaptation of the approach to higher-order Markov order processes.

In addition to that, we aspire to increase the numerical efficiency of the approach. In order to accomplish this, we will reanalyse all the various facets of the approach, such as the VMP inference method or the general filter framework itself.

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