A Novel Approach of Initializing Region-Based Active Contours in Noisy Images by Means of Higher Order Statistics and Dissimilarity

Kevin Ohliger, Torsten Edeler, Stephan Hussmann, Alexandru Paul Condurache, and Alfred Mertins, Senior Member, IEEE

Abstract—In this paper we present two novel methods for initializing region-based active contours. First we use an analysis of higher order statistic attributes of the complete image and the regions of the initial contour candidates. Then we extend this approach by taking the dissimilarity between the initialized regions and the complete image into account. We compare our method with the standard initialization of active contours for different types and degree of noise in synthetic and real images. It is shown that our method outperforms state of the art initializations of active contours considering the total accuracy and Cohen's kappa.

Index Terms - active contours, initialization, noise, kurtosis, dissimilarity, Cohen's kappa, overall accuracy, KR, KDR, higher order statistics, HOS

I. INTRODUCTION

As an important part of image processing segmentation of a scene is still a challenging task and is topic of current research. In 1988 Kass et al. [1] introduced a new segmentation method based on active contour models, also called snakes, using local edge information. An active contour model includes an external energy term which is driven by image data and an internal energy term which makes the active contour behave like an elastic material. Active contours Using edge detectors for the external force were successfully applied for different segmentation applications in [2], [3], [4], [5]. The problems caused by the edge detectors are their sensitivity to noise and discontinuous boundaries in the image as shown in [6]. They were first substituted by region-based approaches in [7]. Related to the Mumford-Shah functional Chan and Vese [6] developed a model assuming that an image is piecewise constant including level sets allowing topological changes of the contours introduced by Osher and Sethian [8]. Shi and Karl [9] introduced a fast level set algorithm without solving partial differential equations.

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of the function \( u \) describing the image. We use a region-based active contour model introduced by [7] defining the energy of a single contour \( i \)
\[
E_{AC_i}(u, C_i, \theta_i) = \frac{\mu}{2} \text{Length}(C_i) - \int_{(x,y) \in R_i} \ln P(u(x,y) | \theta_i) \, dx \, dy. \tag{2}
\]
\( R_i \) is the region enclosed by contour \( C_i \) and \( \theta_i \) contains the statistical parameters describing a feature probability density function of \( R_i \). Length\((C_i)\) is the length of the boundary curve and \( \mu \) a positive weighting factor for the smoothness term. The segmentation result is achieved when the energy of the contour set \( C \) including \( P \) contours \( C_i \) is minimal
\[
\bar{C} = \arg \min_{C \in C} \sum_{i=1}^{P} [E_{AC}(u, C_i, \theta_i) + \lambda]
\tag{3}
\]
\[
= \arg \min_{C \in C} \sum_{i=1}^{P} \left[ \frac{\mu}{2} \text{Length}(C_i) - \int_{(x,y) \in R_i} \ln P(u(x,y) | \theta_i) \, dx \, dy + \lambda \right]. \tag{5}
\]
\( \lambda \) is a positive constant for avoiding oversegmentation and \( \bar{C} \) is the set of final contours. The parameters \( \theta_i \) are estimated based on the sampled image data matrix \( U_0 \) with dimension \( K \times L \). We assume that \( U_0 \) contains \( M \) different objects \( \omega_j \) with a feature (e.g. intensity values) that can be described by a unimodal Gaussian distribution \( \mathcal{N}(\mu_{\omega_j}, \sigma_{\omega_j}^2) \). \( \mu_{\omega_j} \) and \( \sigma_{\omega_j}^2 \) are the feature mean and variance of the respective object. The parameters
\[
\theta_i = [\theta_{i,0}, \theta_{i,1}] = [\mu_{R_i}, \sigma_{R_i}^2]
\tag{6}
\]
of a unimodal Gaussian distribution \( \mathcal{N}(\theta_{i,0}, \theta_{i,1}) \) for Region \( R_i \) are estimated by
\[
\theta_{i,0} = \sum_{j=1}^{M} a_{ji} \cdot \mu_{\omega_j}, \tag{7}
\]
and
\[
\theta_{i,1} = N_{R_i} \left[ \sum_{j=1}^{M} a_{ji} \left( \sigma_{\omega_j}^2 + (\mu_{R_i} - \mu_{\omega_j})^2 \right) \right], \tag{8}
\]
while \( a_{ji} \) describes the ratio of \( \omega_j \) outcomes in region \( R_i \) to complete count of outcomes \( N_{R_i} \) of \( R_i \). The approximated variance (8) leads to good results for \( N_{R_i} > 100 \).

III. IMAGE MODEL

In this section we introduce the image model used. As mentioned in Section II the undistorted image data \( U_0 \) is assumed to include \( M \in [2, K \cdot L] \) different objects
\[
u_0(k,l) = \sum_{j=1}^{M} f_{\omega_j}(k,l), \tag{9}
\]
with \( k = 1, \ldots, K, l = 1, \ldots, L \), and
\[
f_{\omega_j}(k,l) = \begin{cases} F_j(k,l) & k, l \text{ inside of } \omega_j \\ 0 & \text{otherwise}. \end{cases} \tag{10}
\]
\( F_j \) is described by
\[
F_j \sim \mathcal{N}\left(\mu_{\omega_j}, \sigma_{\omega_j}^2\right). \tag{11}
\]
We extend our model (9) with additive noise \( N \) resulting in the noisy image
\[
U = U_0 + N, \tag{12}
\]
while \( N \) is independent identically distributed noise. We concentrate on additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \)
\[
N_n \sim \mathcal{N}(0, \sigma_n^2) \tag{13}
\]
and uniformly distributed noise
\[
N_u \sim \mathcal{U}(0, W), \tag{14}
\]
with \( \mathcal{U} \) described by the probability density function (pdf)
\[
\rho_{\mathcal{U}}(v) = \begin{cases} 1/W & -W/2 \leq v \leq W/2 \\ 0 & \text{otherwise}. \end{cases} \tag{15}
\]
\( W \) is the width of the uniform distribution.

IV. ADAPTATION OF THE INITIAL CONTOUR

This section describes our approach for adaption of the initial contour based on higher order statistics. It is obvious that (5) is strongly influenced by the estimated parameters \( \theta_i \). Initial contours containing more than one object lead to skewed parameters as expressed in (7) and (8) even in a non distorted image \( U_0 \). The general sequence of an automatic selection of regions and their radii based on our two different methods are introduced in the following sections.

A. Initial Contour Adaption Algorithm

We place the center of the region candidates \( R_i \) for initial contours in a regular grid with pitch \( d_0 \). The initial radii are selected to be maximal without generating overlapping regions, \( r_0 = d_0/2 \). The radius for each region is iteratively decreased until the region is expected to include a single object. According to our image model (12) the contained pdf in this case is a unimodal Gaussian pdf distorted by an additive noise. \( R_i \) is a valid initial contour if this is achieved. If a radius is decreased to a fixed minimum \( r_{\text{min}} \), while containing two or more objects, the region is ignored. In Section IV-B and IV-C two methods for analyzing the pdf with respect to the included object count are derived.
B. Kurtosis Driven Radius (KR)

If two or more unimodal Gaussian distributions 
\( N_{1},\ldots,N_{K} \) are overlapping inside one region the type of the mixed distribution \( M \) depends on the parameters of the single distributions. If at least one mean value differs from the other mean values, \( M \) will be a multi-modal Gaussian distribution. Inspired by [21], [22], [23] we choose the kurtosis using the bias corrected formula

\[
\kappa_{\text{corr}}(x) = \frac{L-1}{(L-2)(L-3)} \left[ (L+1) \kappa(x) - 3(L-1) \right] + 3 \quad (16)
\]

with

\[
\kappa(x) = \frac{\sum_{i=1}^{L} (x_i - \mu)^4}{\left( \sum_{i=1}^{L} (x_i - \mu)^2 \right)^2} \quad (17)
\]

as an indicator of multimodal Gaussian distributions with \( L \) samples \( x_i \). As shown in Fig. 1, the kurtosis for a unimodal Gaussian distribution (\( \alpha_1 = 0 \)) is approximately three. For \( \alpha_1 \approx 0.2 \) indicating that 20 percent of the region is filled with another object the kurtosis is also three. This will lead to initial contours with a multi object region. Depending on the global image is also three. This will lead to initial contours with a multi object region. The decision if the region \( R_i \) is a valid initial contour is based on the condition

\[
\kappa_{\text{corr}}(u((x, y) \in R_i)) - 3 \leq T_k \quad , (18)
\]

while \( |:| \) denotes the respective absolute value.

C. Kurtosis and Dissimilarity Driven Radius (KDR)

One main problem of the method introduced in IV-B is the noise sensitivity of the kurtosis as shown in Fig. 2 for Gaussian noise (a) and uniformly distributed noise (b). It is shown that the kurtosis of an image is positive correlated to the additive Gaussian noise energy and negative correlated to the uniformly distributed noise energy. Gaussian noise will lead to initial contours containing invalid regions and uniformly distributed noise will lead to refusing of valid regions. To overcome this we introduce a dissimilarity measure which contains the Wasserstein metric [24] between the estimated cumulated probability density function (cdf) \( F \) of the local region \( R \) and the global image \( U \)

\[
d_{W}(F_R, F_u) = \int_{-\infty}^{\infty} |F_R(v) - F_u(v)| dv \quad . (19)
\]

The selection of the threshold for the dissimilarity \( T_d \) with positive correlation to \( \kappa_{\text{corr}} \) is based on the simulation results shown in Fig. 2 (a) and (b). The condition

\[
d_{W}(F_R, F_u) > T_d \quad (20)
\]

refuses regions with cdfs which are too similar to the global image cdf and hence are dominated by the noise. A disadvantage that arises is that the dissimilarity depends on the mean value of \( R \). Regions with mean values similar to the mean value of \( U \) tend to be rejected. The conditions (18) and (20) are combined for the KDR method.

V. EXPERIMENTAL RESULTS

For evaluation of the methods for contour initializing, the segmentation results for a synthetic image fulfilling our image model (12) and for two real images are compared. The accuracy of the labeled segmentation image \( \tilde{L} \) was measured by the overall accuracy (OA)

\[
OA(\tilde{L}, L) = \sum_{i=1}^{M} \frac{\max (A(\tilde{L} \cap L_i))}{A(L_i)} \quad , (21)
\]

while every labeled region in \( \tilde{L} \) is considered once and \( \max (:\) returns the maximum value. \( A(\cdot) \) is the area of the respective region. \( L \) is the labeled reference containing \( M \) single segmentation labels \( L_i \). In addition to OA, Cohen’s kappa is used

\[
\kappa = \frac{p_o - p_e}{1 - p_e} \quad , (22)
\]

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\[
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\]
Fig. 3 shows the segmentation results and TABLE I the accuracy measurements. The KDR method leads to best result for $U_1$ and $U_3$. In case of $U_2$ the fixed radius selection $r = 5$ performs best with small difference to the KDR method. Choosing $r = 10$ for $U_4$ outperforms the other ones. As shown in the fifth row and column (d) of Fig. 3 the KDR method places initial contours on the border of $\omega_1$ which results in a bad segmentation for this region and leading to lower OA and $\kappa$.

B. Real Images

The evaluation of the segmentation results for real images was made on two different images of the Berkeley segmentation dataset\(^1\). The two images shown in Fig. 4 and 5 were chosen. They are expected to be approximately compatible to the image model (12). The segmentation reference L was also available in the dataset. We choose for KR and KDR methods the parameters $d_0 = 30$ and $r_{\text{min}} = 10$. For the other methods the maximum pitch was $d_0 = 2r$.

The results of the segmentation methods for Gaussian noise with $\sigma_n = 1\text{e-8}$ show small differences in the contour images and also in the OA and $\kappa$ listed in TABLE II. As it can be seen in column (b), (c) of

\(^1\)http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/
Fig. 4. Row 1: Scaled real image 1 to [0,1] with Gaussian noise $\sigma^2_n = 1e-8$ (a), $\sigma^2_n = 4e-2$ (b), and uniformly distributed noise with $W = 0.4$ (c). Row 2: Segmentation result for best fixed radius $r = 30$ (a-c). Row 3: Segmentation result for KR method. Row 4: Segmentation result for KDR method. Row 5: Initial contours of KDR method. For Row 2 to 5 the images are scaled to [0,0.8] in order to emphasize the contour lines.

In TABLE II the segmentation accuracy for real images with additive Gaussian or uniform noise is given.

Table II: Segmentation Accuracy for Real Images with Additive Gaussian or Uniform Noise

<table>
<thead>
<tr>
<th>Img.</th>
<th>Type</th>
<th>$\sigma^2_n$, $W$</th>
<th>Method</th>
<th>OA</th>
<th>$\kappa$</th>
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<td>1</td>
<td>Gaussian</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>KR</td>
<td>0.9245</td>
<td>0.9083</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>KDR</td>
<td>0.9245</td>
<td>0.9083</td>
</tr>
<tr>
<td>1</td>
<td>Gaussian</td>
<td>4e-2</td>
<td>Fixed Radius 30</td>
<td>0.7110</td>
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<td></td>
<td></td>
<td></td>
<td>Fixed Radius 20</td>
<td>0.7088</td>
<td>0.6453</td>
</tr>
<tr>
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<td></td>
<td></td>
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<td>0.5662</td>
<td>0.4730</td>
</tr>
<tr>
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<td></td>
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<td>KR</td>
<td>0.7088</td>
<td>0.6454</td>
</tr>
<tr>
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<td>KDR</td>
<td>0.7221</td>
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</table>

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We have developed two novel methods for shaping initial contours based on higher order statistics and a dissimilarity measure. The KR method varies the radii of possible initial contours by interpreting the kurtosis of the enclosed region and of the complete image. The KDR method extends KR by adding a dissimilarity measure based on the cumulative density functions of local and global regions. The methods were evaluated against standard selections of initial contours including different pitches and radii. The automatic radius adaption of the KDR method lead to better or similar results for low noise energy of Gaussian noise for the synthetic and two real images in terms of overall accuracy and Cohen’s kappa. For uniformly distributed noise the KDR method outperforms the other methods for real images.

B. Future Works

For the KR and KDR method the kurtosis was used as multimodality measure of distributions. The main focus of our actual research is the investigation of further multimodality measures in order to reduce the noise dependency of active contours initialisation methods.

References

Fig. 5. Row 1: Scaled real image 2 to [0,1] with Gaussian noise $\sigma^2 = 1e\text{--}8$ (a), $\sigma^2 = 4e\text{--}2$ (b), and uniformly distributed noise with $W = 0.4$ (c). Row 2: Segmentation result for best fixed radius $r = 30$. Row 3: Segmentation result for KR method. Row 4: Segmentation result for KDR method. Row 5: Initial contours of KDR method. For Row 2 to 5 the images are scaled to [0,0.8] in order to emphasize the contour lines.


