Compression of FFP System Matrix with a Special Sampling Rate on the Lisssajous Trajectory

Marco Maass[a,*], Klaas Bente[b], Mandy Ahlborg[b], Hanne Medimagh[b], Huy Phan[a], Thorsten M. Buzug[b], and Alfred Mertins[a]

[a] Institute for Signal Processing, University of Lübeck, Germany
[b] Institute of Medical Engineering, University of Lübeck, Germany
[*] Corresponding author, email: maass@isip.uni-luebeck.de

INTRODUCTION The tracer based imaging method magnetic particle imaging (MPI) allows reconstructing the distribution of superparamagnetic iron oxid nanoparticles [1]. For the calculation of the spatial particle concentration $\boldsymbol{c}$ from the voltage signal $\boldsymbol{u}$, a system-matrix based reconstruction is widely used. The system matrix $\boldsymbol{S}$, which maps between the particle concentration and voltage signal, has to be known [2]. Since the system matrices can be very large in size and thus consume a huge amount of memory in workspace, a compression method was introduced [3]. The idea was to transform the system matrix via well-known transforms into another space where it becomes sparse.

In this paper, we study the relationship between the sampling pattern and the compressibility of the system matrix and introduce a new compression method for MPI system matrices using a field-free point (FFP) on a Lissajous trajectory.

MATERIAL AND METHODS The signal equation in time-domain is described by $\boldsymbol{u} = \boldsymbol{S}\boldsymbol{c}$, where $\boldsymbol{u} \in \mathbb{R}^N$, $\boldsymbol{S} \in \mathbb{R}^{N\times M}$ and $\boldsymbol{c} \in \mathbb{R}_+^M$. With the discrete Fourier transform matrix $\boldsymbol{\mathcal{F}}$ we obtain the frequency-domain representation $\widehat{\boldsymbol{u}} = \boldsymbol{\mathcal{F}}\boldsymbol{u} = \boldsymbol{\mathcal{F}}\boldsymbol{S}\boldsymbol{c} = \widehat{\boldsymbol{S}}\boldsymbol{c}$, which is the standard representation in MPI. In this work, we consider the two-dimensional case for the field-of-view (FOV). In [3], the rows of $\widehat{\boldsymbol{S}}$ were transformed with a transformation matrix $\boldsymbol{T}$ with respect to the dimensions of the FOV. Hence, the signal equation was re-written as $\widehat{\boldsymbol{u}} = \widehat{\boldsymbol{S}}\boldsymbol{T}\boldsymbol{T}^{-\boldsymbol{1}}\boldsymbol{c} = \widehat{\boldsymbol{S}}_{\boldsymbol{T}}\boldsymbol{c}_{\boldsymbol{T}}$. The work in [3] also showed that the discrete Chebychev transform (DTT) and discrete cosine transform of type two (DCT-II) are able to compress system matrices significantly.

For the compression scheme proposed in this work, the frequency ratio is chosen to be rational with $f_x/f_y = N_y/N_x = N_y/(N_y-1)$ where $N_x, N_y \in \mathbb{N}$. We can show that, if we sample the trajectory equidistantly in time at $N_s = k \cdot N_x \cdot N_y$ sampling points in one period, where $k \in \mathbb{N}$, we obtain k separable Euclidian-like grids. The sampling points are shown for $N_y = 5$ and two different k in Fig. 1. Some equivalent observation results were recently also confirmed in [4]. In this paper, we show that the orthogonal transform on the FOV should have symmetric and antisymmetric basis functions to result in a maximally sparse representation of the system matrix. In fact, for simulated system matrices, the use of a spatial transform with the above mentioned symmetry properties results in a large number of coefficients in $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ being exactly zero. For measured system matrices, these values are approximately zero. While the DCT-II and the DTT automatically satisfy the symmetry requirements, they are not yet optimal to compress the system matrix. We show that a further compression can be achieved by applying a secondary orthogonal transform to the matrix $\widehat{\boldsymbol{S}}_{\boldsymbol{T}}$ that can be composed of optimized rotation matrices.

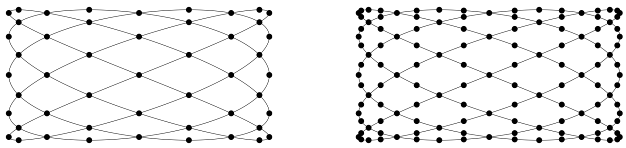

Figure 1: Sampling pattern for a Lissajous trajectory with the ratio $f_x/f_y = 4/5$ and $k = 4$ (left) and $k = 8$ (right).

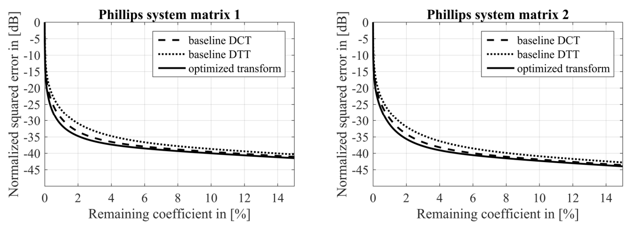

Figure 2: Normalized squared error as function of the percentage of remaining coefficients after hard-thresholding in the range of 0% to 15%. The highest possible error is 0 dB.

For testing, we use the system matrices from the Philips dataset [2]. The frequency ratio in this dataset is $f_x/f_y = 33/32$, and the above mentioned time-domain sampling condition was originally fulfilled when this matrix was acquired. In a second step, several frequencies have been deleted so that only the first 1268 frequency components were available.

RESULTS As can be seen in Fig. 2, our method is performing with a better compression ratio in terms of normalized mean squared error than the standard approach by [3] on the Philips system matrix dataset. One can see that gains of up to 2 dB can be achieved.

CONCLUSIONS We showed that the number of samples per Lissajous trajectory has to obey a certain sampling rule and that the applied spatial transform should obey certain symmetries to ensure sparsity for the transform coefficients. Experimentally, we were able to verify that under these conditions, better compacting transforms can be found than the DCT or DTT.

ACKNOWLEDGEMENTS This work was supported by the German Research Foundation under Grant No. ME 1170/7-1 and BU 1436/7-1.

REFERENCES

[1] B. Gleich and J. Weizenecker. *Nature*, 435(7046):1217—1217, 2005. doi:10.1038/nature03808.
[2] T. Knopp and T. M. Buzug. Springer, Berlin/Heidelberg, 2012. doi: 10.1007/978-3-642-04199-0.
[3] J. Lampe et al, *Physics in Medicine and Biology*, 57(4):1113—1134, 2012.
doi:10.1088/0031-9155/57/4/1113.
[4] W. Erb et al, *Numerische Mathematik* , 2015. doi: 10.1007/s00211-015-0762-1