Lifting Scheme – an Alternative to Wavelet Transforms for Real Time Applications

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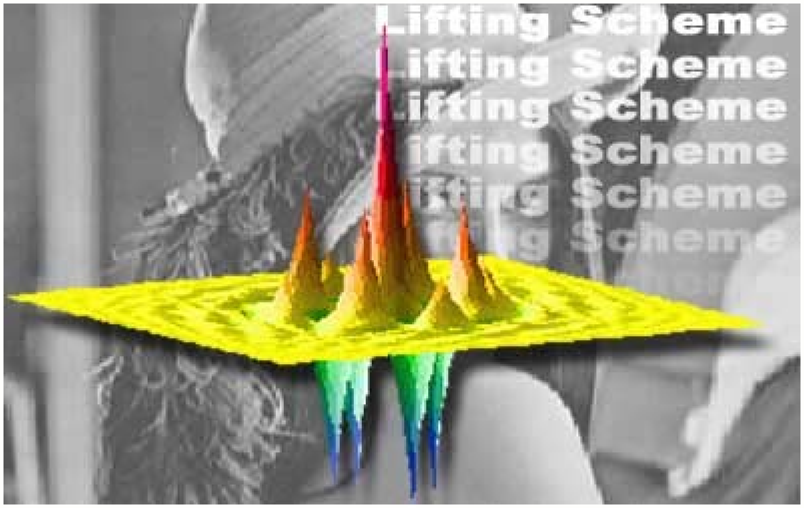

Lifting Scheme – an Alternative to Wavelet Transforms for Real Time Applications

Wavelet transformation became quite recently an extremely useful tool for all practitioners doing high-performance signal processing and is described in detail elsewhere [1, 2]. In its discrete form, signals are decomposed by so called compact supported base functions, which may be time- and frequency limited. Contrary to the Fourier transformation, this leads to a high precision in time AND frequency resolution. This high precision is achieved by both applying a high- and low-pass filter, representing the scaling function of the mother-wavelet, to the signal and successively downsample both results [3].

Decomposition levels are arranged in a pyramidal scheme, where from one decomposition level to the next the low pass sampled results are kept for further processing, whereas the high-pass filtered results are stored as "details". The required filtering is quite frequently done by "quadrature mirror filters" (see GlobalDSP magazine, August 2003). As long as no resulting coefficients in each level are dropped or distorted, the signal is perfect reconstructable.

Fig. 1 (on next page) shows the filter bank for N =3. In each step the signal is decomposed by applying complementary FIR-filters to *aj* , i.e. a high-pass g' and a low pass h', which are determined by the selected wavelet. The results of both filter operations are subsampled by a factor of two, leading to subbands *dj+1* and *aj+1*. Note, that the number of coefficients in *dj+1* and *aj+1* are equal to the number of coefficients in *aj* .

Even though DWT is an effective way to decompose a signal into its subbands and thus being able to perform de-noising and compression on the signal, some process steps seem to behave to generous with the limited resources available in real time signal processing.

Fortunately, a group of wavelet-forerunners found out, that the above mentioned pyramidal proecess based on QMF's is not the only way to perform the wavelet decomposition [4-7]. By diving more deeply into the mathematical theory behind multiscale mathematics, they suggested a different method called the "lifting scheme". This scheme replaces the usual FIR filtering and downsampling process in each level by splitting

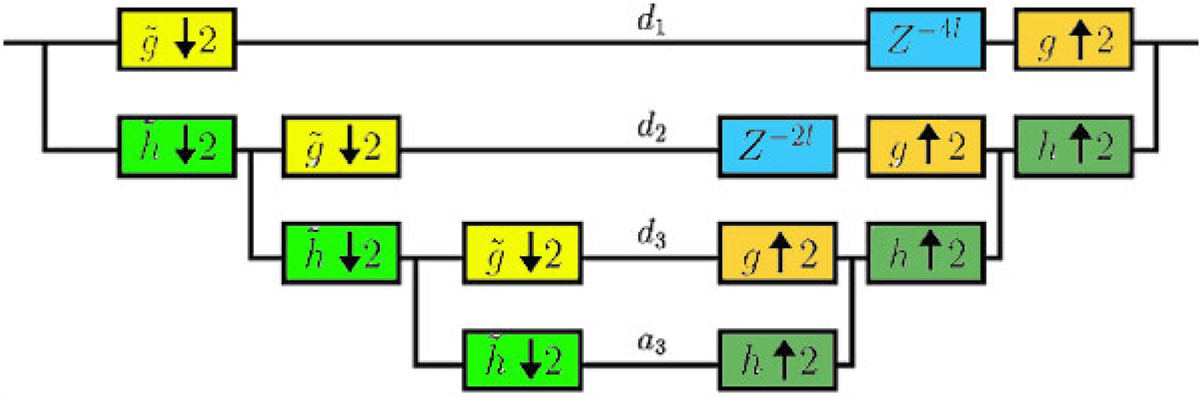

Figure 1: Filter bank with three levels

the signal in two series of values with odd and even running indices. In a simple case, only the odd values are used to predict (P) remaining even values, which leads to coefficients originating from the resulting estimation errors to the real even values ("dual lifting"). Those coefficients are now in the following "primal lifting" used to adjust and update (U) the original odd values themselves. Figure 2 shows the schematics of one such lifting step and the successive reconstruction step (as part of a pyramidal WT decomposition).

Given the lifting scheme, how exactly do we choose the operations for prediction and update? In fact we want to implement certain already well known Wavelets (e.g. Daubechies Wavelets) which are usually available as a pair of high and low pass FIR filter.

On the way from a FIR filter pair (h(z),g(z)) to the lifting steps we first construct the 2 by 2 poly-phase matrix which entries are filters consisting of the odd and even numbered filter coefficients of h and g, respectively. The polyphase matrix performs exactly the operation which is shown in Figure 3 (left side), but using the polyphase matrix we have already saved half of the operations compared to a straight

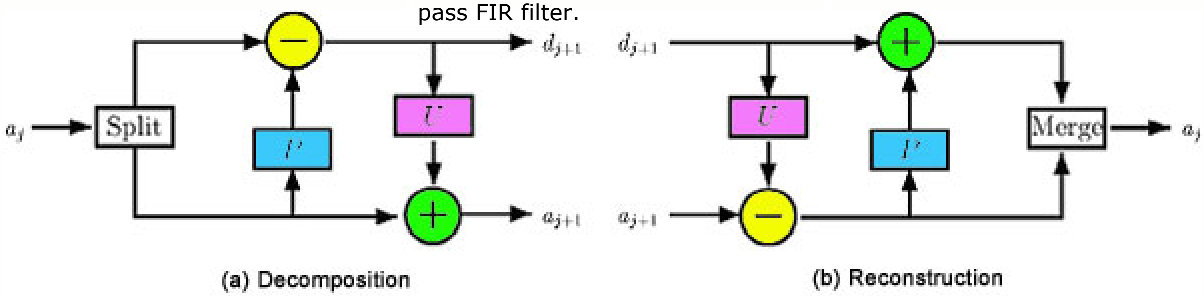

Figure 2: Basic structure of the lifting scheme: a) Decomposition and b) Reconstruction.
a) A "split" operator divides the signal in series of "odd" and "even" signal indices. The resulting values are then used to predict ("dual lifting", P) or update ("primal lifting", U) the corresponding values.
b) a simple reversal of all signs leads to the corresponding reconstruction lifting steps.

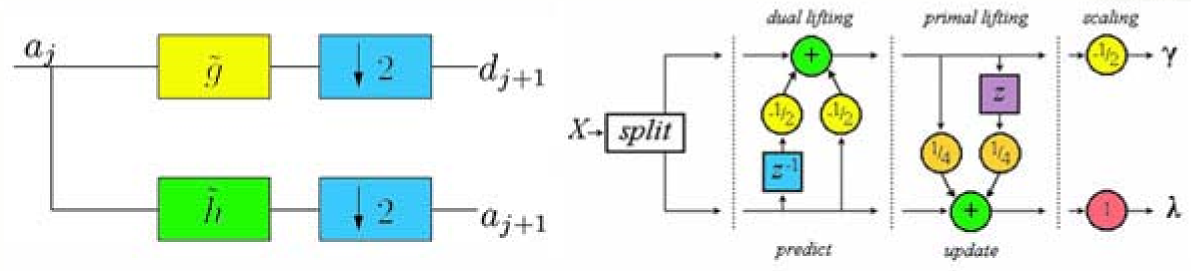

Figure 3: Exemplary steps to produce corresponding detail and approximation coefficients by either half a quadrature filter bank or by a lifting scheme with two lifting steps.

implementation of the scheme in Figure 3 (left).

As the next step we decompose the polyphase matrix into a product of lifting step matrices using the Euclidean algorithm for Laurent polynomials. It was shown in [4] that such a decomposition always exists and that we obtain even faster implementations of a certain wavelet decomposition.

Note, that the decomposition of the polyphase matrix leads to several update and predict steps. However, each step is simple, i.e. comparable to a filter with two coefficients.

The lifting scheme has the perfect reconstruction property by design: Whatever (!) the operations P and U do to their inputs, we can always reconstruct the original signal by reconstruction scheme.

As a consequence we can easily modify the lifting scheme such that it maps integer inputs onto integer coefficients by introduction of a rounding operation after P and U, while P and U still perform floating point operations.

The lifting scheme has two extremely useful advantages over regular DWT by FIR filters, which speed up real-time calculations: No need for extrinsic memory allocation and results shifting exists, since the lifting calculations are performed “in place”.

The lifting may furthermore even be performed with integer values only. All in all, this alternative way of WT out-performs with higher order, more complex mother-wavelets the DWT by a factor of 4. This is at least in our extremely constrained condition for multichannel signal processing on 128 channels a more than significant advantage.

With all these advantages, why do not more practitioners use this method to perform the WT online? One reason may be the bulky theory to represent a beautiful mother-wavelet by a series of small matrizes and lifting steps.

The other may be that there is no unique way of performing exactly this step. However, there is no real need to dive deeply into these mathematical details, since Clemens Valens provides on his webpage a simple program written for Matlab (The Mathworks, Inc.) [6], which produces all possible poly-phase matrices for a given mother-wavelet by the Euclidian algorithm for polynoms.

Unfortunately, there may be a couple of thousand solutions for a simple wavelet like a Daubechies-6 and many more for higher order wavelets. Still, there are ways to chose one of these solutions and produce subsequently an Assembler code file representing exactly this lifting solution. That way, we are now able to perform computationally complex tasks like wavelet de-noising and neuronal spike recognition on 128 channels sampled with 40kHz on TMS C6x DSPs [8-10].

The appropriate source code and C files including examples and descriptions of this realtime analysis are available here: (http://web.isip.uni-luebeck.de/~hofmann/Paper/2003/Wavelet_Lifting/)

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