Multiresolution Magnetic Particle Imaging of Vessel Structures with Support Detection

C. Droigk[a,*], M. Maass[a], P. Koch[a], A. Möller[a], and A. Mertins[a]

[a] *Institute for Signal Processing, Universität zu Lübeck, Lübeck, Germany*
[*] *Corresponding author, email: droigk@isip.uni-luebeck.de*

Abstract: In Magnetic Particle Imaging, the distribution of a tracer is measured. In most practical cases, the tracer is only present at few areas and most of the image consists of background. Therefore, it seems promising to detect the support of the tracer and include this knowledge in the reconstruction process. In this work, a method is proposed which performs support detection in a multiresolution analysis and includes certainty about the estimation via a weighting in the optimization process. The results show that this procedure improves the structural similarity index.

I. Introduction

Magnetic Particle Imaging (MPI) is a medical imaging method that measures the nonlinear magnetization of super-paramagnetic iron-oxide particles (SPIOs), which are used as tracer [1]. As a tracer-based imaging method, it is promising for studying vessel structures. Due to the high spatial resolution of MPI, vessels can be detected even at a lower resolution level. Furthermore, vessel images contain a relatively high amount of background pixels, because the tracer is only located inside the vessels. In this work, we combine the multiresolution reconstruction based on wavelet decomposition from [2] with a level-wise tracer support detection, which serves as prior information for the next level. We estimate the probability with which a certain pixel belongs to the foreground on the basis of the reconstruction at the lower resolution and use this estimation as a weighting in the reconstruction process for the next level. There are a few works with similar ideas. In [3], the idea was formulated to exclude background in the reconstruction. Prior information was also used in [4] in the reconstruction, but it was obtained from magnetic resonance images. In [5], a joint reconstruction and segmentation was performed on positron emission tomography data without a multiresolution analysis.

II. Material and Methods

II.I Methods

The standard method for MPI image reconstruction is the minimization of the objective function

$$\frac{1}{2}\|Sc-u\|_2^2+\lambda R(c), \qquad (1)$$

where $S \in \mathbb{C}^{N,M}$ is the system matrix in the Fourier domain, $u \in \mathbb{C}^M$ contains the Fourier coefficients of the measured voltage signal, $c \in \mathbb{R}_+^N$ denotes the SPIOs' concentration, which shall be reconstructed, and $R(c)$ is a regularization term which is weighted with $\lambda > 0$. A common choice is the Tikhonov regularization $R(c)=\|c\|_2^2$. To include prior knowledge about the SPIO distribution, the regularization term is modified and the search space of the optimizer is restricted. With an estimation of the foreground probability for each pixel, we can formulate the problem as

$$\hat{c} = \arg\min_c \frac{1}{2}\|Sc-u\|_2^2+\lambda\|Wc\|_2^2, \ \text{s.t.}\ c\in K \qquad (2)$$

where $W \in \mathbb{R}^{N,N}$ is a diagonal matrix with the weighting w_{ii} of the concentration c_i. The w_{ii} can be chosen large at positions which are probably background pixels and low for foreground pixels. However, the w_{ii} should not become zero, because this could lead to unbounded values of c_i. If we have an estimation for the foreground probability $P^{fg} \in [0,1]^N$ for every pixel, we can choose K in (2) as $K=\{c\in\mathbb{R}_+^N: c_i=0, \text{if } P_i^{fg}=0\}$, which reduces the search space. Given the probability estimation P^{fg} we can set $w_{ii}=1+C(1-P_i^{fg})$ with constant $C>0$ specifying the influence of the probability estimation. This choice ensures $w_{ii}>0$ for all i. To obtain an estimation of the foreground, a soft segmentation of the tracer concentration at a coarser level can be performed. This estimation can be resized to the actual size by bilinear interpolation and used as prior information. This can be repeated until the finest resolution level is reached. The procedure is visualized in Fig. 1. There are different methods for soft segmentation. Due to its simplicity, we chose a random walker (RW) segmentation algorithm [6]. For solving the problem (2) the fast iterative shrinkage threshold algorithm [7] is used. For the general step of the algorithm, the proximal operator for $R(c)=\lambda\|Wc\|_2^2$ with constraint $c\in K$ is needed. With W being a diagonal matrix and $\mathbb{1}_K$ the indicator function of K, it yields

$$\operatorname{prox}_{R(c)}(x)=\mathbb{1}_K(x)\frac{x}{1+2\lambda\operatorname{diag}(W^2)}, \qquad (3)$$

where division is performed pointwise.

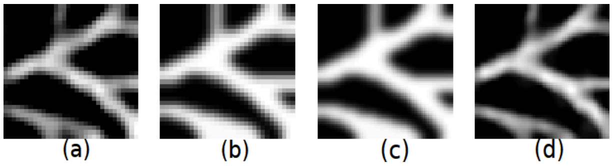

Figure 1: *Reconstruction and support detection procedure. (a) Image reconstruction at level K. (b) Estimated foreground probability at level K. (c) Upsampled foreground estimation. (d) Image reconstruction at level K-1 using (c).*

II.II Simulation

For evaluation, simulated data are used. The MPI scanner simulation uses the Langevin model of paramagnetism and a Lissajous FFP-trajectory with frequency ratio $f_x/f_y = 32/33$. The approach was tested on two vessel phantoms shown in Fig. 2 (left). The phantoms have a size of 250 × 250 pixels with value 0 (black) at background and 1 (white) and 0.4 (dark gray) in the structures. We used a 9/7 wavelet decomposition in four levels. The constant C was set to 15. The algorithm needs foreground and background seeds, which were chosen automatically as the smallest 40 % of values for the background seed and the highest 20 % values for the foreground seed. For each phantom, a simulated voltage signal with signal-to-noise ratios (SNRs) between 5 and 35 dB was used for reconstruction. A multilevel reconstruction without foreground detection was performed for comparison, which is referred to as baseline. The structural similarity index (SSIM) and the mean absolute error (MAE) of the reconstructions were computed. An SSIM near 1 indicates highly similar structures, while an MAE near to 0 means low errors in the concentration values.

III. Results

In Fig. 2, the reconstructed SPIO distributions for an SNR of 5 dB for the two approaches are shown. The experiments reveal that the proposed method suppresses the noise in the background effectively. For Phantom A less blurred edges are observed. In Fig. 3, the SSIM and MAE for the different SNRs are shown. The parameter λ which produced the best result for SSIM or MAE among the tested values was chosen. The SSIM value of the level-wise segmentation method outperforms the baseline on all phantoms and for all SNRs. The MAE is better for low SNRs using the support detection approach.

IV. Discussion

The proposed multiresolution segmentation method improves the SSIM of the reconstructions. For low SNRs an improvement of the MAE can be seen as well. This is due to the deletion of noise in the background and a tendency to less blurring around the edges of the structures. Although some structures in phantom B have low intensities, they are detected as foreground. It is thinkable that small, isolated structures with low intensities could be removed by the method. For low SNRs those structures are hardly reconstructed by the baseline method either.

V. Conclusions

The multilevel segmentation method provides a reconstruction with improved SSIM. Especially at signals with low SNR it can help to enhance image quality. The results are promising and let one expect that they might further improve with a more sophisticated segmentation method.

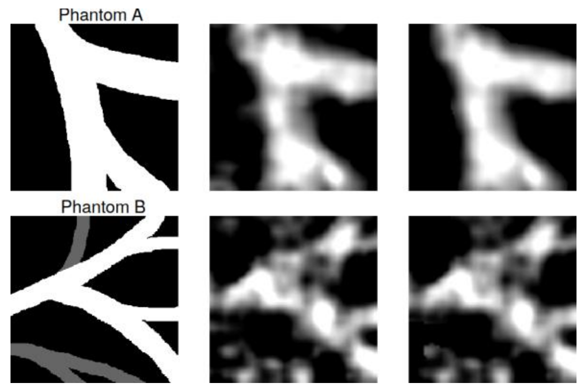

Figure 2: *Reconstructions of the phantoms (left) with best MAE value of the baseline method (middle) and the multilevel segmentation method (right) at an SNR of 5 dB.*

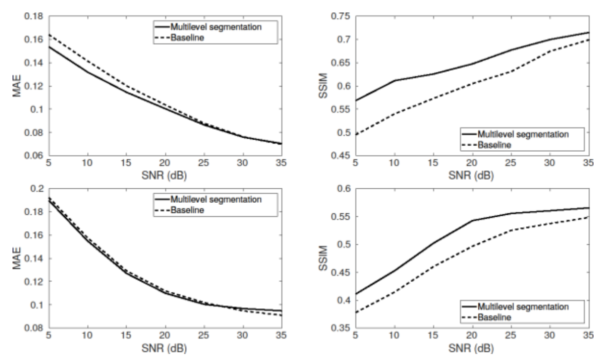

Figure 3: *MAE (left) and SSIM (right) of the phantom reconstructions A (top) and B (bottom).*

AUTHOR'S STATEMENT

This work was supported by the German Research Foundation under grant number ME 1170/7-1. Authors state no conflict of interest.

REFERENCES

[1] B. Gleich and J. Weizenecker. Tomographic imaging using the nonlinear response of magnetic particles. *Nature*, 435(7046):1217-1217, 2005. doi: 10.1038/nature03808.
[2] M. Maass, C. Mink and A. Mertins. Joint multiresolution magnetic particle imaging and system matrix compression. *Int. J. Magn. Part. Imaging*, 4(2), 2018, doi:10.18416/ijmpi.2018.1811002.
[3] H. Siebert, M. Maass, M. Ahlborg, T. M. Buzug and A. Mertins. MMSE MPI Reconstruction Using Background Identification. *Int. Workshop Magn. Part. Imaging,* 58, 2016.
[4] C. Bathke, T. Kluth, C. Brandt and P. Maass. Improved image reconstruction in magnetic particle imaging using structural a priori information. *Int. J. Magn. Part. Imaging*. 3(1), 2017.
[5] Storath, A. Weinmann, J. Frikel and M. Unser. Joint image reconstruction and segmentation using the Potts model. *Inverse Problems*, 31(2), 2015. doi: 10.1088/0266-5611/31/2/ 025003.
[6] L. Grady. Random walks for image segmentation. *IEEE Trans.Pattern Anal. Mach. Intell*., 18(11): 1768-1783, 2006. doi: 10.1109/tpami.2006.233.
[7] A. Beck and M. Teboulle. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM J Imaging Sci*., 2(1): 183-202, 2009. doi: 10.1137/080716542.