Design of Filterbank Transceivers for Dispersive Channels with Arbitrary-Length Impulse Response

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Abstract

This paper addresses the joint design of transmitter and receiver for multichannel data transmission over dispersive channels. The transmitter is assumed to consist of FIR filters and the channel impulse response is allowed to have arbitrary length. The design criterion is the maximization of the information rate between transmitter input and receiver output under the constraint of a fixed transmit power. A link to minimum mean squared error designs for a similar setting is established. The proposed algorithm allows a straightforward transmitter design and generally yields a near-optimum solution for the transmit filters. Under certain conditions, the exact solution for the globally optimal transmitter is obtained.

1 Introduction

The joint design of transmitter and receiver for data transmission over dispersive channels has attracted numerous researchers, as it has the potential to yield very high throughput without the need of costly algorithms on the receiver side, such as maximum likelihood sequence estimation with the Viterbi algorithm. The process of shaping the transmit signal and/or introducing redundancy based on the knowledge of the channel is also known as precoding. Salz [1] provided a first solution to the joint transmit/receive filter design problem, but it required the filters to have support within the first Nyquist zone $[-1/2T, 1/2T]$. Yang and Roy proposed an algorithm for the design of precoders that use excess bandwidth to introduce redundancy [2]. Their method required an iteration to find the optimum solution. Xia studied the existence of redundant precoders that allow a perfect inversion of FIR channels with FIR receivers [3]. The effects of noise were not considered in [3]. Direct solutions to the joint design problem for the case of block transforms with a sufficiently long guard interval to avoid interblock interference (IBI) were provided in [4–6]. The optimality criteria considered in [4] were the zero forcing (ZF) and minimum mean squared error (MMSE) criteria. In [5] and [6] the maximization of mutual information between transmitter and receiver was studied, using results derived in [7]. A drawback of the block transforms of [4–6] is that the length of the guard interval needs to be at least equal to the channel order. This is the same problem as with the well-known DMT and OFDM techniques [8,9]. To cope with longer channel impulse responses one can increase the length of the guard interval, but this will decrease the efficiency, as less data symbols can be transmitted. Increasing both the length of the guard interval and the number of subchannels allows one to maintain a desired bandwidth efficiency, but this strategy also has its limits, because the delay between transmitter and receiver may become unacceptably high. Li and Ding provided a direct solution to the problem of minimizing the mean squared error (MSE) between transmitter input and receiver output under the power constraint for arbitrary channel lengths with overlapping blocks [10]. However, their solution generally yields IIR transmit filters, which restricts the practical use of their exact solution. An FIR approximation of the technique in [10] was provided in [11]. Finally, transmitter design methods for the case where decision feedback receivers are employed have been proposed in [7, 12, 13].

This paper addresses the design of FIR precoders for the case where the channel impulse response has arbitrary length. Note that this configuration is of significant interest for practical applications, because real-world channel impulse responses may become extremely long and the use of sufficiently long guard intervals, as required for DMT, OFDM, or the methods in [4–6], may be prohibitive due to delay constraints. During transmitter optimization an approximation is used that allows us to simplify the objective function and obtain a straightforward solution. For $L \leq N - M$, where L is the channel order, M is the number of subchannels, and N is the upsampling factor in the transmitter, the algorithm yields the exact optimum solutions of [5, 6], and for $L > N - M$ it leads to near optimum solutions.

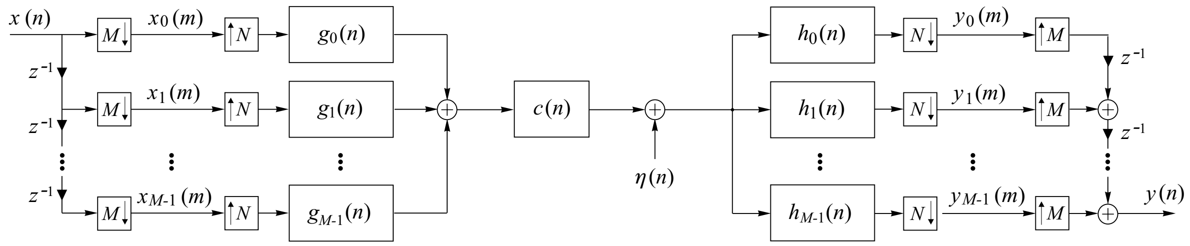

Figure 1: Redundant precoder.

The paper is organized as follows. Section 2 describes the input-output relationships of the considered transmit/receive system. Section 3 then addresses the maximization of the information rate through the choice of optimal transmit and receive filters. Also a link to MMSE designs for similar settings is established. Section 4 demonstrates the properties of the proposed algorithm in several examples, and finally Section 5 gives some conclusions.

Notation. Vectors and matrices are printed in boldface. The superscripts $\{\cdot\}^T$, $\{\cdot\}^H$, $\{\cdot\}^+$ denote transposition, Hermitian transposition, and the pseudoinverse. The determinant and trace of a matrix are denoted as $|\cdot|$ and tr $\{\cdot\}$, respectively. $E\{\cdot\}$ is the expectation operation.

2 System Description

A block diagram of the considered system is depicted in Figure 1. The input stream $x(m)$ is split into M parallel streams which are then upsampled by a factor of $N \geq M$ and fed into the M transmit filters with impulse responses $g_k(n),\ k = 0, 1, \ldots, M-1$. The channel is described by its impulse response $c(n)$ and an additive, data independent, zero-mean, stationary, Gaussian noise process $\eta(n)$. The receive signal is filtered with the analysis filters $h_k(n),\ k = 0, 1, \ldots, M-1$ and subsampled by N to yield the parallel output data $y_k(m)$. Finally, a parallel-to-serial conversion yields the output sequence $y(n)$.

For further analysis it is advantageous to decompose the filters into their polyphase components and to describe the system as a multiple-input multiple-output (MIMO) system as depicted in Figure 2. The input vector at time m is given by $\boldsymbol{x}(m) = [x_0(m), x_1(m), \ldots, x_{M-1}(m)]^T$ with $x_k(m) = x(mM-k)$. Accordingly, the output process $\boldsymbol{y}(m)$ is defined as $\boldsymbol{y}(m) = [y_0(m), \ldots, y_{M-1}(m)]^T$. The transmit filter bank can be described via its $N \times M$ polyphase matrix [14]

$$\boldsymbol{G}(z) = \begin{bmatrix} G_{00}(z) & \ldots & G_{M-1,0}(z) \\ \vdots & & \vdots \\ G_{0,N-1}(z) & \ldots & G_{M-1,N-1}(z) \end{bmatrix} \quad (1)$$

where $G_{k,\ell}(z)$ is the ℓth polyphase component of the kth transmit filter, given by

$$G_{k,\ell}(z) = \sum\nolimits_n g_k(nN+\ell)\, z^{-n}. \quad (2)$$

Alternatively, $\boldsymbol{G}(z)$ may be expressed as $\boldsymbol{G}(z) = \sum_n \boldsymbol{G}_n z^{-n}$ with $[\boldsymbol{G}_n]_{\ell,k} = g_k(nN+\ell)$ where $[\boldsymbol{G}_n]_{\ell,k}$ denotes the element of $[\boldsymbol{G}_n]$ at position ℓ, k.

The polyphase matrix of the receiver filter bank is given by

$$\begin{aligned} \boldsymbol{H}(z) &= \sum\nolimits_n \boldsymbol{H}_n z^{-n} \\ &= \begin{bmatrix} H'_{00}(z) & \ldots & H'_{0,N-1}(z) \\ \vdots & & \vdots \\ H'_{M-1,0}(z) & \ldots & H'_{M-1,N-1}(z) \end{bmatrix} \end{aligned} \quad (3)$$

with

$$\begin{aligned} H'_{k,\ell}(z) &= \sum\nolimits_n h_k(nN+N-1-\ell)\, z^{-n}, \\ [\boldsymbol{H}_n]_{k,\ell} &= h_k(nN+N-1-\ell). \end{aligned} \quad (4)$$

The channel can be described via the pseudo-circulant $N \times N$ matrix

$$\boldsymbol{C}(z) = \begin{bmatrix} C_0(z) & z^{-1}C_{N-1}(z) & \ldots & z^{-1}C_1(z) \\ C_1(z) & C_0(z) & \ldots & z^{-1}C_2(z) \\ \vdots & & \ddots & \vdots \\ C_{N-1}(z) & C_{N-2}(z) & \ldots & C_0(z) \end{bmatrix} \quad (5)$$

with $C_\ell(z) = \sum_n c(nN+\ell)\, z^{-n}$. Alternatively, $\boldsymbol{C}(z)$ can be written as a polynomial of matrices:

$$\boldsymbol{C}(z) = \sum\nolimits_k z^{-k}\, \boldsymbol{C}_k. \quad (6)$$

The often desired (zero forcing) property

$$\boldsymbol{y}(n) = \boldsymbol{x}(n - n_0) \quad (7)$$

is obtained in the noise free case if $\boldsymbol{H}(z)$ and $\boldsymbol{G}(z)$ are chosen such that the perfect reconstruction (PR) condition

$$\boldsymbol{H}(z)\, \boldsymbol{C}(z)\, \boldsymbol{G}(z) = z^{-n_0+1} \boldsymbol{I}_{M\times M} \quad (8)$$

holds. Conditions to satisfy (7) for a given channel $c(n)$ are for example discussed in [3, 4].

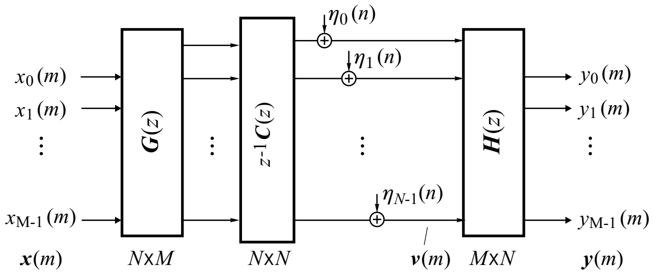

Figure 2: Redundant precoder in polyphase (MIMO) representation.

3 Maximizing Information Rate

In this section we address the problem of maximizing the information rate through the choice of the transmit and receive filters. We will first consider a straightforward matrix model, similar to block transforms, and will show for this model that the mutual information can be expressed via the error covariance matrix of MMSE receive filters. Using this fact, an algorithm for determining optimal FIR transmit filters is presented.

3.1 A General Expression for Mutual Information

The mutual information between a block of input symbols, $\boldsymbol{x}$, and a block of output symbols, $\boldsymbol{y}$, of a transceiver is defined as $I_0(\boldsymbol{x};\boldsymbol{y}) = H(\boldsymbol{x}) - H(\boldsymbol{x}|\boldsymbol{y})$ where $H(\boldsymbol{x})$ is the entropy of $\boldsymbol{x}$ and $H(\boldsymbol{x}|\boldsymbol{y})$ is the conditional entropy of $\boldsymbol{x}$ given $\boldsymbol{y}$ [15]. We define a normalized mutual information as

$$I(\boldsymbol{x};\boldsymbol{y}) = \frac{1}{N}[H(\boldsymbol{x}) - H(\boldsymbol{x}|\boldsymbol{y})] \qquad (9)$$

where N is the upsampling factor in Figure 1. The length of $\boldsymbol{x}$ is M with $M \leq N$, and the length of $\boldsymbol{y}$ will be defined as needed. It is known that $I(\boldsymbol{x};\boldsymbol{y})$ becomes maximal if $\boldsymbol{x}$ is Gaussian [15], and therefore we will assume Gaussian processes henceforth. For this case it was shown in [7] that

$$I(\boldsymbol{x};\boldsymbol{y}) = \frac{1}{N}\log_2\left(\frac{|\boldsymbol{R}_{xx}|}{|\boldsymbol{R}^{\perp}_{x|y}|}\right) \qquad (10)$$

with

$$\boldsymbol{R}^{\perp}_{x|y} = \boldsymbol{R}_{xx} - \boldsymbol{R}_{xy}\boldsymbol{R}^{-1}_{yy}\boldsymbol{R}_{yx} \qquad (11)$$

and $\boldsymbol{R}_{xx} = E\left\{\boldsymbol{x}\boldsymbol{x}^H\right\}$, $\boldsymbol{R}_{xy} = \boldsymbol{R}^H_{yx} = E\left\{\boldsymbol{x}\boldsymbol{y}^H\right\}$, $\boldsymbol{R}_{yy} = E\left\{\boldsymbol{y}\boldsymbol{y}^H\right\}$.

We now consider the model

$$\boldsymbol{y} = \boldsymbol{H}[\boldsymbol{C}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{n}] \qquad (12)$$

where the matrices $\boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}$ describe the transmitter, channel, and receiver, respectively, and vector $\boldsymbol{n}$ describes additive noise. At this point, no assumptions are made about the size of vectors and matrices in (12) and the type of noise. With (12) one obtains for $\boldsymbol{R}^{\perp}_{x|y}$

$$\begin{aligned}\boldsymbol{R}^{\perp}_{x|y} = {} & \boldsymbol{R}_{xx} - \boldsymbol{R}_{xx}\boldsymbol{G}^H\boldsymbol{C}^H\boldsymbol{H}^H \times \\ & \times[\boldsymbol{H}(\boldsymbol{C}\boldsymbol{G}\boldsymbol{R}_{xx}\boldsymbol{G}^H\boldsymbol{C}^H + \boldsymbol{R}_{nn})\boldsymbol{H}^H]^{-1} \times \\ & \times \boldsymbol{H}\boldsymbol{C}\boldsymbol{G}\boldsymbol{R}_{xx},\end{aligned} \qquad (13)$$

with $\boldsymbol{R}_{nn} = E\left\{\boldsymbol{n}\boldsymbol{n}^H\right\}$. By using the pseudoinverse of $\boldsymbol{R}^{\perp}_{x|y}$ given by

$$\begin{aligned}(\boldsymbol{R}^{\perp}_{x|y})^{+} = {} & \boldsymbol{R}^{+}_{xx} + \\ & + \boldsymbol{G}^H\boldsymbol{C}^H\boldsymbol{H}^H[\boldsymbol{H}\boldsymbol{R}_{nn}\boldsymbol{H}^H]^{-1}\boldsymbol{H}\boldsymbol{C}\boldsymbol{G}\end{aligned} \qquad (14)$$

the quantity $I(\boldsymbol{x};\boldsymbol{y})$ can be alternatively expressed as

$$I(\boldsymbol{x};\boldsymbol{y}) = \frac{1}{N}\log_2\left|\boldsymbol{R}_{xx}\,(\boldsymbol{R}^{\perp}_{x|y})^{+}\right|. \qquad (15)$$

Note that the expression (14) for $(\boldsymbol{R}^{\perp}_{x|y})^{+}$ includes the shaping of the transmit signal with matrix $\boldsymbol{G}$ and the influence of the receive filters in matrix $\boldsymbol{H}$. A similar expression for mutual information has been derived in [7], but for the simpler model $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{n}$ with $\boldsymbol{n}$ being white noise. Using the results of [7] and a model similar to (12), but without possible interblock interference, a related expression has also been obtained in [5].

3.2 Incorporating the Filterbank Model

Now let the model (12) describe the filterbank transceiver of Section 2 with $\boldsymbol{x} := \boldsymbol{x}(m)$ and $\boldsymbol{y} := \boldsymbol{y}(m - n_0)$. The columns of matrix $\boldsymbol{G}$ are the transmit filter impulse responses, and the channel matrix $\boldsymbol{C}$ has the structure

$$\boldsymbol{C} = \begin{bmatrix} c(0) & 0 & 0 & 0 & \dots & 0 \\ c(1) & c(0) & 0 & 0 & \dots & 0 \\ c(2) & c(1) & c(0) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \end{bmatrix}. \qquad (16)$$

The size of $\boldsymbol{C}$ depends on the lengths of the transmit filters and the channel. $\boldsymbol{C}$ may even be of infinite dimension, and similarly, the vector $\boldsymbol{v} = \boldsymbol{C}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{n}$ observed at the channel output may be of infinite length. However, both $\boldsymbol{x}$ and $\boldsymbol{y}$ are of length M. The noise process

$\boldsymbol{n}$ contains the additive channel noise and the IBI from other data blocks.

In the following we show that the optimal receive matrix $\boldsymbol{H}$ has the structure

$$\boldsymbol{H} = \boldsymbol{X}\boldsymbol{G}^H\boldsymbol{C}^H\boldsymbol{R}_{nn}^{-1} \qquad (17)$$

with an arbitrary, full-rank $M \times M$ matrix $\boldsymbol{X}$. Depending on $\boldsymbol{X}$ one obtains, for example, the ZF or MMSE receive filters. Inserting (17) into (14) and rearranging the obtained expression yields

$$(\boldsymbol{R}_{x|y}^{\perp})^{+} = \boldsymbol{R}_{xx}^{+} + \boldsymbol{G}^H\boldsymbol{C}^H\boldsymbol{R}_{nn}^{-1}\boldsymbol{C}\boldsymbol{G}. \qquad (18)$$

Note that (18) is independent of $\boldsymbol{X}$. Obviously, $(\boldsymbol{R}_{x|y}^{\perp})^{+}$ according to (18) is the same as the matrix $(\boldsymbol{R}_{x|v}^{\perp})^{+}$, which relates to the conditional entropy $H(\boldsymbol{x}|\boldsymbol{v})$ based on the observation $\boldsymbol{v}$. Because of $H(\boldsymbol{x}|\boldsymbol{y}) \leq H(\boldsymbol{x}|\boldsymbol{v})$, we can conclude that any matrix $\boldsymbol{H}$ of the form (17) maximizes the mutual information. Thus, due to the structure of $\boldsymbol{H}$ in (17) this means that the optimal receive filters are "matched filters", given by the term $\boldsymbol{G}^H\boldsymbol{C}^H\boldsymbol{R}_{nn}^{-1}$, followed by an arbitrary, full-rank matrix operation $\boldsymbol{X}$. Through the choice of $\boldsymbol{X}$ one can obtain, for example, the optimal zero forcing and MMSE solutions.

Interestingly, the matrix $(\boldsymbol{R}_{x|y}^{\perp})$ is the same as the error correlation matrix

$$\boldsymbol{R}_{x|y}^{\perp} := \boldsymbol{R}_{ee} = E\left\{(\boldsymbol{y}-\boldsymbol{x})(\boldsymbol{y}-\boldsymbol{x})^H\right\}$$

for the case of linear MMSE estimation of $\boldsymbol{x}$ from the noisy observation $\boldsymbol{v}$.[1] This observation has also been made in [7]. For the filterbank transceivers considered in this papers it means that we can concentrate on minimizing the determinant of the error correlation matrix in the presence of an MMSE receive filterbank. To simplify the notation we assume white channel noise with variance σ_η^2 and white data $x(n)$ with variance σ_x^2. The incorporation of nonwhite data and noise processes is straightforward.

For further derivations, the expression (18) for $(\boldsymbol{R}_{x|y}^{\perp})^{+}$ is not very convenient, as it contains the inverse correlation matrix of the noise which is comprised of channel noise and IBI. Knowing that we need the error correlation matrix of MMSE estimation we can alternatively use the expression obtained in [11] for MMSE precoders:

$$\boldsymbol{R}_{ee} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sigma_x^2 \left[\boldsymbol{I}_{M\times M} + \right. \\ \left. + \frac{\sigma_x^2}{\sigma_\eta^2}\boldsymbol{G}^H(e^{j\omega})\left[\sum\nolimits_k \boldsymbol{\mathcal{R}}_{cc}(k)e^{-j\omega k}\right]\boldsymbol{G}(e^{j\omega})\right]^{-1} d\omega \qquad (19)$$

where

$$\boldsymbol{\mathcal{R}}_{cc}(k) = \sum\nolimits_\ell \boldsymbol{C}_\ell^H\boldsymbol{C}_{\ell+k}. \qquad (20)$$

[1] Introductions to linear estimation theory can be found in [16].

3.3 Using FIR transmit filters

To minimize the transmitter complexity and system delay, we assume transmit filters of length N where N is the upsampling factor in Figure 1. For this filter length we have $\boldsymbol{G}(z) = \boldsymbol{G}_0$ and obtain

$$\boldsymbol{R}_{ee} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sigma_x^2 \left[\boldsymbol{I}_{M\times M} \right. \\ \left. + \frac{\sigma_x^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\left[\sum\nolimits_k \boldsymbol{\mathcal{R}}_{cc}(k)e^{-j\omega k}\right]\boldsymbol{G}_0\right]^{-1} d\omega. \qquad (21)$$

The next step is to approximate (21) by a simpler expression. Because the summation terms for $k \neq 0$ in (21) relate to IBI we choose $\boldsymbol{G}_0$ from a subspace such that the terms $\boldsymbol{G}_0^H\boldsymbol{\mathcal{R}}_{cc}(k)\boldsymbol{G}_0$ for $k \neq 0$ become so small that they can be neglected in (21). To determine a suitable subspace for the choice of $\boldsymbol{G}_0$ we employ an iterative procedure based on the singular value decomposition (svd). We do not explicitly formulate a basis for the required subspace, and rather consider a projection $\boldsymbol{P}$ that projects onto the required subspace.

The algorithm is as follows:

Step 1: Let $\boldsymbol{P} = \boldsymbol{I}_{N\times N}$

Step 2: Compute the svd's

$$\boldsymbol{A}_k\boldsymbol{\Sigma}_k\boldsymbol{B}_k^H = \boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(k)\boldsymbol{P}$$

for all $k \neq 0$ for which $\boldsymbol{\mathcal{R}}_{cc}(k) \neq \mathbf{0}$.

Step 3: Determine the largest singular value for $k \neq 0$ and denote it as σ_{max}. Assuming that σ_{max} is contained in matrix $\boldsymbol{\Sigma}_K$ denote the corresponding column of $\boldsymbol{A}_K$ as $\boldsymbol{a}$.

Step 4: If rank$(\boldsymbol{P}) > M$ and $\sigma_{max} > 0$ set

$$\boldsymbol{P} := \left[\boldsymbol{I}_{N\times N} - \boldsymbol{a}\boldsymbol{a}^H\right]\boldsymbol{P}$$

and go back to Step 2. Otherwise, end the algorithm.

When incorporating the projection matrix $\boldsymbol{P}$, the error correlation matrix can be approximated by

$$\tilde{\boldsymbol{R}}_{ee} = \sigma_x^2\left[\boldsymbol{I}_{M\times M} + \frac{\sigma_x^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}\boldsymbol{G}_0\right]^{-1}, \qquad (22)$$

and the normalized mutual information can thus be approximated as

$$\bar{I}(\boldsymbol{x};\boldsymbol{y}) = \frac{1}{N}\log_2(|\boldsymbol{M}|) \qquad (23)$$

with

$$\boldsymbol{M} = \left[\boldsymbol{I}_{M\times M} + \frac{\sigma_x^2}{\sigma_\eta^2}\boldsymbol{G}_0^H\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}\boldsymbol{G}_0\right]. \qquad (24)$$

According to Hadamard's inequality [15], $\boldsymbol{M}$ must be diagonal in order to maximize $|\boldsymbol{M}|$ under the transmit power constraint

$$\sigma_x^2 \operatorname{tr}\left\{\boldsymbol{G}_0\boldsymbol{G}_0^H\right\} = N \cdot P_0. \tag{25}$$

This means that the columns of $\boldsymbol{G}_0$ have to be scaled eigenvectors of $\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P}$. We now consider the eigendecompositions

$$\boldsymbol{P}^H\boldsymbol{\mathcal{R}}_{cc}(0)\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H \tag{26}$$

and

$$\boldsymbol{G}_0\boldsymbol{G}_0^H = \boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^H \tag{27}$$

with

$$\boldsymbol{\Lambda} = \operatorname{diag}\left[\lambda_1, \ldots, \lambda_N\right] \tag{28}$$

and

$$\boldsymbol{Q} = \operatorname{diag}\left[q_1, \ldots, \sigma_N\right] \tag{29}$$

where the eigenvalues λ_i are assumed to be sorted such that $\lambda_i \geq \lambda_i + 1$. Note that some of the eigenvalues λ_i may be zero and that only the first M values $q_1, \ldots, q_M$ are non-zero. Using (26) and (27) the mutual information $\bar{I}(\boldsymbol{x};\boldsymbol{y})$ according to (23) and (24) can be rewritten as

$$\bar{I}(\boldsymbol{x};\boldsymbol{y}) = \frac{1}{N}\sum_{i=1}^{M}\log_2(1+\frac{\sigma_x^2}{\sigma_\eta^2}\lambda_i q_i) \tag{30}$$

A standard Lagrange optimization, similar to [5, 7], yields

$$q_i = \max(c - \frac{\sigma_\eta^2}{\sigma_x^2\lambda_i}, 0) \tag{31}$$

where c is to be determined from the power constraint (25). As one can see in (31), the optimal values q_i obey the waterpouring distribution. Assuming that M is chosen such that $q_i, i = 1, \ldots, M$ are nonzero, the transmit filters finally become

$$\boldsymbol{G}_0 = \bar{\boldsymbol{U}} \operatorname{diag}\left[\sqrt{q_1}, \ldots, \sqrt{q_M}\right] \tag{32}$$

where $\bar{\boldsymbol{U}}$ contains the M eigenvectors that belong to the largest eigenvalues $\lambda_1, \ldots, \lambda_M$. A comparison with the solution in [11] shows that maximizing the information rate and minimizing the overall MSE leads to the same transmit filters, but with different power loading factors $q_i,\ i = 1, \ldots, M$. Moreover, it is straightforward to show that if the channel order L is smaller or equal to $N - M$ we have $\bar{I}(\boldsymbol{x};\boldsymbol{y}) = I(\boldsymbol{x};\boldsymbol{y})$, and the proposed algorithm yields the solutions of [5, 6].

4 A Design Example

We demonstrate the performance of the precoder design algorithm using a simple example where significant IBI between adjacent data blocks occurs. The chosen parameters are $L = 6$, $N = 16$, $M = 14$, and the E_b/N_0 ratio at the receiver input is set to $20dB$. The channel impulse response is $c(n) = [1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1]$. Note that all channel zeros lie on the unit circle of the z-plane. The frequency response of the channel is depicted in Figure 3, together with the transmit power spectra for the following two precoder design methods: (i) the MMSE precoder of [11] and (ii) the precoder maximizing information rate proposed in this paper. The comparison between the two power spectra shows that the MMSE precoder tends to spend power in frequency bands where the channel gain is low, whereas the precoder maximizing information rate reduces the transmit power for such frequencies.

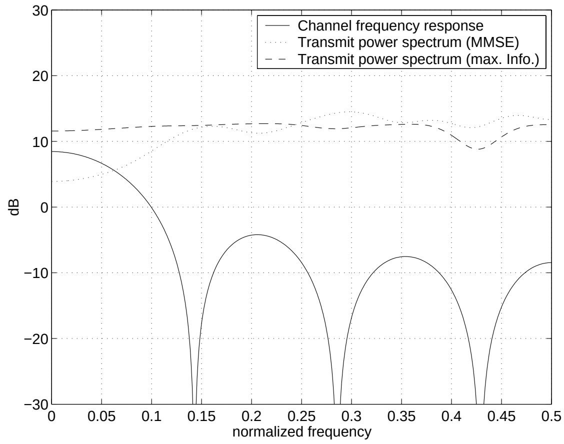

Figure 3: Channel frequency response and transmit power spectra.

Figure 4 shows the obtained SNR's at the receiver output for the two design methods. One can see that maximizing the information rate yields several subchannels with very good SNR and a few with poor SNR. The MMSE design, on the other hand, tries to uphold all SNR's in order to minimize the MSE. The obtained normalized information rates are 3.49 bit/symbol for the MMSE design and 3.84 bit/symbol when maximizing $\bar{I}(\boldsymbol{x},\boldsymbol{y})$.

When reducing the number of subchannels to $M = 10$, all IBI vanishes, and the design method becomes equivalent to the ones in [5, 6]. However, the maximum normalized mutual information is only 3.47 bit/symbol for this case, which shows that allowing IBI has the potential to improve performance compared to block transmission.

5 Conclusions

A method for the joint design of transmitter and receiver for data transmission over dispersive channels has been presented. The proposed method maximizes the information rate and can treat the practically important case where the transmitter is FIR and the channel

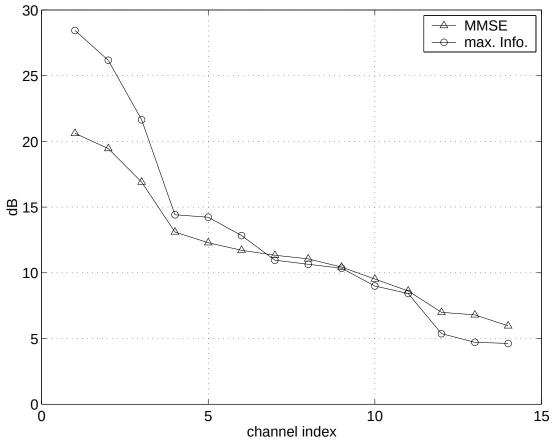

Figure 4: Signal to noise ratios it subchannels at the receiver output.

has arbitrary length. This allows for low latency transmission over dispersive channels. Design examples have confirmed the effectiveness of the design method.

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