

MEMORY TRUNCATION RECEIVERS FOR TRANSMULTIPLEXERS

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Abstract

In this paper, the design of optimal receive filter banks for transmultiplexer-based data transmission over frequency selective channels is investigated. A new design strategy based on the principle of memory truncation, rather than equalization, is presented. Through the receive filters, each subchannel is truncated to a certain length, and the actual data detection is then carried out via low-complexity, independently operating Viterbi detectors. Design examples are presented for high-speed transmission over copper wires. The examples show that memory truncation allows significant performance improvements over MMSE equalization.

Keywords: Transmultiplexer, memory truncation, Viterbi detector, dispersive channel, filter banks.

1 Introduction

The performance of transmission systems based on discrete multitone (DMT) modulation [1–3] or orthogonal frequency division multiplex (OFDM) [4] degrades rapidly when the length of the channel impulse response exceeds the length of the guard interval, which is introduced to cope with non-ideal channels. As a result of an insufficient guard interval, intersymbol interference (ISI) will occur. One possibility to cope with longer channel impulse responses is to increase the length of the guard interval, but this will decrease the efficiency, as less data symbols can be transmitted. Increasing both the length of the guard interval and the number of subchannels allows one to maintain a de-

sired bandwidth efficiency, but this strategy also has its limits. For example, the delay between transmitter and receiver may become unacceptably high. Also, the hardware requirements increase with an increasing number of subchannels. Finally, channels which can be regarded as slowly time-varying when the number of subchannels is low may turn into fast time-varying ones if the number of subchannels and thus the lengths of the transmit and receive filters are significantly increased.

In this paper, new methods for the design of optimal receive filter banks in multichannel transmission systems are proposed. The results are presented for a multirate filter bank framework, which gives a common description of a variety of transmission techniques [5]. The solutions apply to DMT [3, 6], OFDM [4], coded-OFDM [7], transmultiplexers [8,9], and other transmission techniques where the transmit signal is created as a weighted linear combination of basis sequences with the data symbols being the weights. Even code division multiplex (CDMA) [10, 11] can be seen as a multirate filter bank. Fig. 1 shows the general structure of the transmit/channel/receive model used in this paper. To simplify the notation, we will refer to this system as a transmultiplexer. Depending on the actually wanted modulation technique, the upsampling factor, N , the number of subchannels, M , and the impulse responses $g_k(n)$ and $h_k(n)$ are chosen.

Various solutions to the problem of reducing ISI in transmultiplexing systems through channel equalization have been proposed [5, 10, 12–19]. Most of them are based on minimizing the mean squared error (MSE)

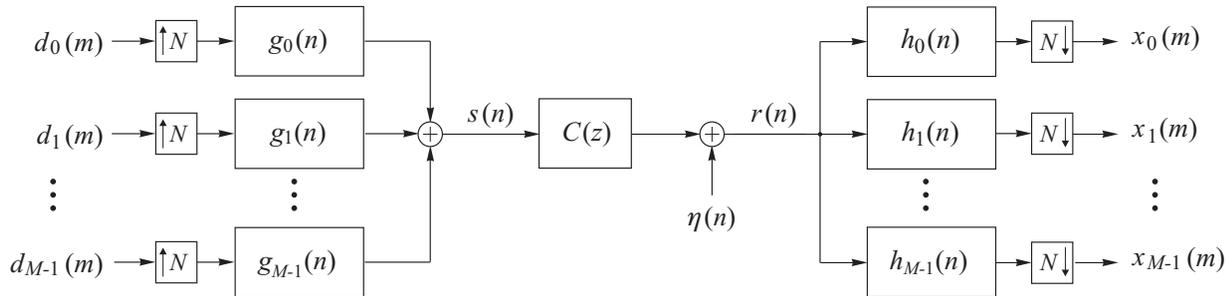


Fig. 1. Multirate discrete-time transmitter/channel/receiver model.

between the sent data and the equalizer output, either using a general MSE or a zero forcing (ZF) concept. Decision feedback equalizers (DFE) have been considered in [17]. Minimum mean squared error (MMSE) and ZF solutions with a joint design of receiver and transmitter have been proposed in [5, 16, 18]. Such a joint design can be useful in cases where communication takes place in both directions. In this paper, we concentrate on the optimal receiver design, thus addressing cases where the transmitter is fixed. The methods proposed in this paper are extensions of the technique in [19] to the design of entire receive filter banks for the oversampled case. The design criterion is based on the idea of memory truncation [20, 21], where the receiver does not try to fully equalize the channel and leaves a residual system in the data path. In the optimum, the MSE between the equalizer output and a filtered version of the input data sequence is minimized. The final data detection then takes place via a Viterbi detector which needs to consider only the residual impulse responses. The lengths of the residual filters can be chosen arbitrarily and will typically be a few taps, thus allowing the use of low-complexity Viterbi detectors. The advantage of memory truncation over equalization is that critical channel zeros (e.g. zeros close to or even on the unit circle) need not be equalized, so that the problem of noise amplification through the equalizer can be avoided.

Note that for DMT transmission, memory truncation has also been proposed in a different form where the channel memory is shortened to the length of the guard interval prior to the DFT analysis in the receiver [22, 23]. In the present paper, however, memory truncation is incorporated as a property of the receive filters,

and we can even treat cases where no guard interval is introduced at all.

The paper is organized as follows. In Section 2 the input-output relations for the multirate system in Fig. 1 are outlined. Section 3 addresses the design of optimal receive filter banks. Results are discussed in Section 4, and finally some conclusion are given in Section 5.

Notation: The superscript T denotes transposition of a vector or matrix. The superscripts $*$ and H denote complex conjugation and conjugate transposition ($\mathbf{r}^H = [\mathbf{r}^*]^T$), respectively. \mathbf{I} is an identity matrix of appropriate size. $E\{\}$ denotes the expectation operation, and $\delta_{i,k}$ is the Kronecker symbol.

2 Input-Output Relations

We consider the system in Fig. 1 which depicts the general transmitter/channel/receiver model. The input sequences $d_k(m)$, $k = 0, 1, \dots, M-1$ are typically created through a series-to-parallel conversion of a single data sequence $d(m)$ in the form $d_k(m) = d(mM - k)$, $k = 0, 1, \dots, M-1$. In other words, they are polyphase components of the sequence $d(n)$. As shown in Fig. 1, the data sequences $d_k(m)$ are upsampled by a factor of N and then fed into the M respective synthesis filters with impulse responses $g_k(n)$, $k = 0, 1, \dots, M-1$. The sum of the filtered signals finally forms the transmit signal

$$s(n) = \sum_{k=0}^{M-1} \sum_{m=-\infty}^{\infty} d_k(m) g_k(n - mN). \quad (1)$$

Typically, the filters $g_k(n)$ are chosen to be frequency selective, so that each data sequence $d_k(m)$ is transmitted in a distinct frequency band. This is for example the

case in DMT, OFDM and cosine-modulated transmultiplexers as in [14].

To make certain that the input data can be recovered at least theoretically from the transmit signal $s(n)$, the upsampling factor N must be chosen such that $N \geq M$ [9]. In many practical systems $N > M$ is used, which means that the transmitter introduces redundancy. This redundancy can be utilized in the receiver for enhancing the performance in the presence of frequency selective channels.

Considering a time invariant channel, the receive signal is given by

$$r(n) = \left[\sum_{m=-\infty}^{\infty} c(m) s(n-m) \right] + \eta(n), \quad (2)$$

where $\eta(n)$ is an additive, data independent noise process and $c(n)$ is the channel impulse response. The noise is assumed to be zero mean and wide-sense stationary.

On the receiver side, the signal $r(n)$ is fed into the analysis filter bank, as shown in Fig. 1, and the filter output signals are subsampled by a factor of N to form the final output signals

$$x_k(m) = \sum_{n=0}^{L_h-1} h_k(n) r(mN-n) \quad (3)$$

with $k = 0, 1, \dots, M-1$. In (3), L_h is the length of the receive filters. Combining (1), (2) and (3) we get the input-output relation

$$\begin{aligned} x_k(m) = & \sum_{n=0}^{L_h-1} \sum_{\mu=-\infty}^{\infty} \sum_{i=0}^{M-1} \sum_{\ell=-\infty}^{\infty} h_k(n) c(\mu) d_i(\ell) \\ & \cdot g_i(mN-n-\mu-\ell N) \\ & + \sum_{n=0}^{L_h-1} h_k(n) \eta(mN-n). \end{aligned} \quad (4)$$

for $k = 0, 1, \dots, M-1$.

Under ideal conditions where the analysis and synthesis filters of the transmultiplexer form a perfect reconstruction (PR) filter bank and where the channel is noise free and ideal (i.e. $\eta(n) = 0 \forall n$ and $c(n) = \delta_{n,0}$) the transmit/receive system allows us to recover the data $d_k(m)$ without error, but with an overall delay of m_0 samples:

$$x_k(m) = d_k(m-m_0). \quad (5)$$

The PR conditions for the filter bank itself are

$$\sum_{n=0}^{L_h-1} h_k(n) g_i(mN-n) = \delta_{i,k} \delta_{m,m_0} \quad (6)$$

with $i, k = 0, \dots, M-1$. A practical problem is that even transmitter/receiver systems satisfying (6) will be unable to perfectly recover the data if a non-ideal channel is introduced. Thus, the channel should be taken into account when designing the receive filter bank. Methods for this will be discussed in the next section.

3 Design of Optimal Receive Filter Banks

In this section, we derive methods for the design of optimal receive filter banks. For this we define an error signal as the difference between the receiver output signals $x_k(m)$ and filtered versions of the data sequences:

$$\begin{aligned} e_k(m) = & \sum_{j=0}^{L_h-1} h_k(j) r(mN-j) \\ & - \sum_{i=0}^{L_p-1} p_k(i) d_k(m-m_0-i), \end{aligned} \quad (7)$$

$$k = 0, \dots, M-1.$$

The optimality criteria for the design of the M receive filters are the MSEs given by

$$Q_k = E \left\{ |e_k(m)|^2 \right\}, \quad k = 0, \dots, M-1, \quad (8)$$

which are to be minimized under the energy constraints,

$$\sum_{n=0}^{L_p-1} |p_k(n)|^2 = 1, \quad k = 0, 1, \dots, M-1. \quad (9)$$

The constraints (9) are needed to avoid the trivial solution $h_k(n) = 0, p_k(n) = 0$.

Note that the error measure (7) is different from the MSE as defined for conventional MSE equalizers [10, 12–15]. The idea behind the proposed approach is to truncate the channel memory and not to delete it completely. The impulse responses $p_k(m)$ are to be understood as residual impulse responses of arbitrarily chosen length L_p . During the receiver design process both the optimal residual systems $p_k(m)$ and receive filters $h_k(n)$ need to be found through minimization of (8).

Because of the existence of residual systems $p_k(m)$, minimizing (8) does, in general, not result in an equalization of the channel. Even if $Q_k = 0$ there will be a remaining ISI between L_p consecutive data samples in each of the subchannels.

With analysis filters designed through the minimization of (8) the overall system can be modeled with little error as a set of M independent channels with

$$x_k(m) = \left[\sum_{i=0}^{L_p-1} p_k(i) d_k(m - m_0 - i) \right] + \eta'_k(m), \quad k = 0, \dots, M - 1. \quad (10)$$

The modified noise processes $\eta'_k(m)$ contain the filtered and subsampled original noise and all modeling errors made by simplifying the real system to the form (10).

To recover the data, the signals $x_k(m)$, $k = 0, \dots, M - 1$ are fed into M independently operating Viterbi detectors which have to consider the respective channels $p_k(m)$, $k = 0, \dots, M - 1$. Since the lengths of these channels are chosen arbitrarily, one can choose lengths which result in a manageable computational cost for the Viterbi detectors while giving a low noise variance at the detector inputs. Clearly, the longer the systems $p_k(m)$ are, the smaller the modeling errors in (10) and thus the smaller the variances $E\{|\eta'(m)|^2\}$ are. For $L_p = 1$ the Viterbi detectors degenerate to simple threshold detectors, at the expense of an increased noise variance compared to cases where $L_p > 1$.

Note that in the special case of $L_p = 1$, Eq. (8) states a standard MSE criterion, and the optimized analysis filters $h_k(n)$ can be regarded as MMSE equalizers. Then the proposed solution becomes equivalent to other known MMSE solutions [10, 12–15].

To obtain a compact formulation of the objective function, we now introduce the following vectors:

$$\mathbf{h}_k = [h_k(0), \dots, h_k(L_h - 1)]^T \quad (11)$$

$$\tilde{\mathbf{r}}(m) = [r(mN), \dots, r(mN - L_h + 1)]^T \quad (12)$$

$$\mathbf{p}_k = [p_k(0), \dots, p_k(L_p - 1)]^T \quad (13)$$

$$\mathbf{d}_k(m) = [d_k(m), \dots, d_k(m - L_p + 1)]^T \quad (14)$$

We get

$$e_k(m) = \tilde{\mathbf{r}}^T(m) \mathbf{h}_k - \mathbf{d}_k^T(m) \mathbf{p}_k. \quad (15)$$

Using this notation the cost functions $Q_k = E\{|e_k(m)|^2\}$, $k = 0, \dots, M - 1$ can finally be written as

$$Q_k = \mathbf{h}_k^H \mathbf{R}_{rr} \mathbf{h}_k - \mathbf{h}_k^H \mathbf{R}_{rd}^{(k)} \mathbf{p}_k - \mathbf{p}_k^H \mathbf{R}_{dr}^{(k)} \mathbf{h}_k + \mathbf{p}_k^H \mathbf{R}_{dd}^{(k)} \mathbf{p}_k \quad (16)$$

with

$$\mathbf{R}_{rr} = E\{\mathbf{r}^*(m) \mathbf{r}^T(m)\},$$

$$\mathbf{R}_{rd}^{(k)} = [\mathbf{R}_{dr}^{(k)}]^H = E\{\mathbf{r}^*(m) \mathbf{d}_k^T(m - m_0)\},$$

$$\mathbf{R}_{dd}^{(k)} = E\{\mathbf{d}_k^*(m - m_0) \mathbf{d}_k^T(m - m_0)\}.$$

For the sake of simplicity, let us assume that all data sequences $d_k(m)$ are white and have the same variance σ_d^2 . Then the autocorrelation matrices $\mathbf{R}_{dd}^{(k)}$, $k = 0, \dots, M - 1$ are diagonal with diagonal entries σ_d^2 ,

$$\mathbf{R}_{dd}^{(k)} = \sigma_d^2 \mathbf{I}, \quad (17)$$

and (16) simplifies to

$$Q_k = \mathbf{h}_k^H \mathbf{R}_{rr} \mathbf{h}_k - \mathbf{h}_k^H \mathbf{R}_{rd}^{(k)} \mathbf{p}_k - \mathbf{p}_k^H \mathbf{R}_{dr}^{(k)} \mathbf{h}_k + \sigma_d^2 \mathbf{p}_k^H \mathbf{p}_k. \quad (18)$$

We now consider the minimization of (18) with respect to \mathbf{p}_k and \mathbf{h}_k under the energy constraints (9). To derive the optimal filters we first derive the optimal vector \mathbf{h}_k given a fixed residual system \mathbf{p}_k . From $\partial Q_k / \partial \mathbf{h}_k = \mathbf{0}$ with Q_k as in (18), we get

$$\mathbf{h}_k^{(\text{opt})} = \mathbf{R}_{rr}^{-1} \mathbf{R}_{rd}^{(k)} \mathbf{p}_k. \quad (19)$$

Substituting $\mathbf{h}_k^{(\text{opt})}$ into (18) results in

$$Q_k = -\mathbf{p}_k^H [\mathbf{R}_{rd}^{(k)}]^H \mathbf{R}_{rr}^{-1} \mathbf{R}_{rd}^{(k)} \mathbf{p}_k + \sigma_d^2 \mathbf{p}_k^H \mathbf{p}_k, \quad (20)$$

which now is to be minimized with respect to \mathbf{p}_k under the constraint (9). This yields the eigenvalue problems

$$\left[\sigma_d^2 \mathbf{I} - [\mathbf{R}_{rd}^{(k)}]^H \mathbf{R}_{rr}^{-1} \mathbf{R}_{rd}^{(k)} \right] \mathbf{p}_k = \lambda_k \mathbf{p}_k, \quad k = 0, 1, \dots, M - 1, \quad (21)$$

which are essentially similar to the one in [20] for the single-channel case. The optimal vectors \mathbf{p}_k are the eigenvectors that belong to the respective smallest eigenvalues λ_k , $k = 0, \dots, M - 1$.

The receive filters designed according to the method described above minimize the error measures Q_k under

the energy constraint and thus maximize the signal-to-noise ratios (SNRs) at the filter outputs. Since the filter output signals, together with the residual systems, are fed into the Viterbi detectors, the algorithm maximizes the SNRs as seen by the Viterbi detectors.

4 Results

To demonstrate the performance of the proposed algorithms, we first consider the transmission of data over telephone lines in an ADSL/VDSL related setting [24]. Fig. 2 shows the channel impulse response considered in this example. It is assumed that the channel noise is comprised of near and far end crosstalk as well as white gaussian noise, resulting in the total power spectral density depicted in Fig. 3. We consider the use of a cosine-modulated filter bank for creating the transmit signal, which is an interesting alternative to blockwise DFTs as in DMT. In [14, 25] it was shown that such filter bank based transmultiplexers offer greater potential than blockwise DFTs because of their longer impulse responses and better frequency selectivity. However, they need equalization on the receiver side. In the present example, the transmit signal is synthesized via a 16-band cosine-modulated filter bank with ELT (extended lapped transform, [26]) prototype. Pulse amplitude modulation is used to create a real-valued transmit signal. Note that this is essentially the same concept as in [14, 25].

Figs. 4 and 5 show the signal-to-noise ratios within the different bands at the equalizer output for several configurations. In all cases the lengths of the receive filters are chosen as $L_h = 128$. We first look at the results depicted in Fig. 4. In this case, all bands are loaded with the same input power σ_d^2 . This means that the transmultiplexer is critically sampled and that no redundancy (e.g. in form of a guard interval) is introduced. The comparison of the three curves in Fig. 4 shows that, especially for the low-frequency channels, memory truncation ($L_p > 1$) significantly outperforms MMSE equalization ($L_p = 1$).

A significantly better performance of all methods under consideration is obtained if the first frequency band remains unloaded. Results are depicted in Fig. 5. This strategy has been proposed in [25] as a possibility to introduce redundancy. Leaving out a particular

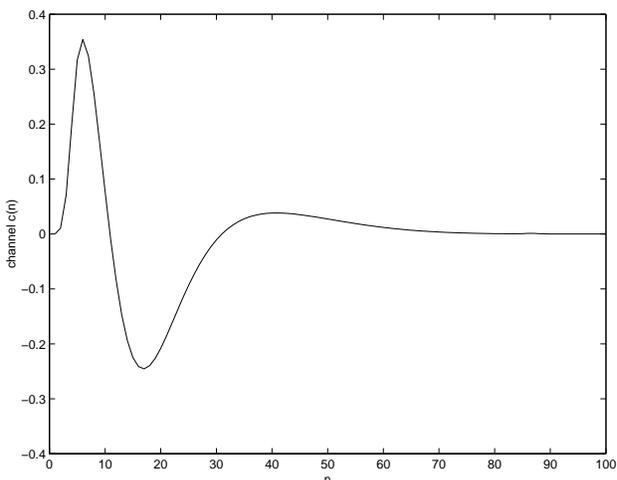


Fig. 2. Transmission channel impulse response.

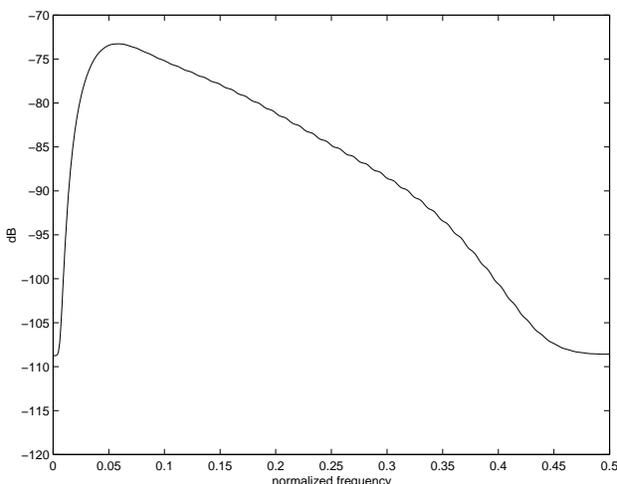


Fig. 3. Noise power spectral density.

band has two effects. Firstly, the system becomes over-sampled, which means that the transmitter introduces redundancy in form of excess bandwidth. Secondly, the receive filters do not need to suppress crosstalk from the dropped channel and have more freedom to equalize their own data paths. As the results in Fig. 5 show, almost all channels gain from the fact that the first band has been left out. Experiments have shown that leaving out another band while keeping band zero does not yield a comparable improvement. This suggests that band zero is extremely critical in this case. When comparing the three curves in Fig. 5, we see that memory

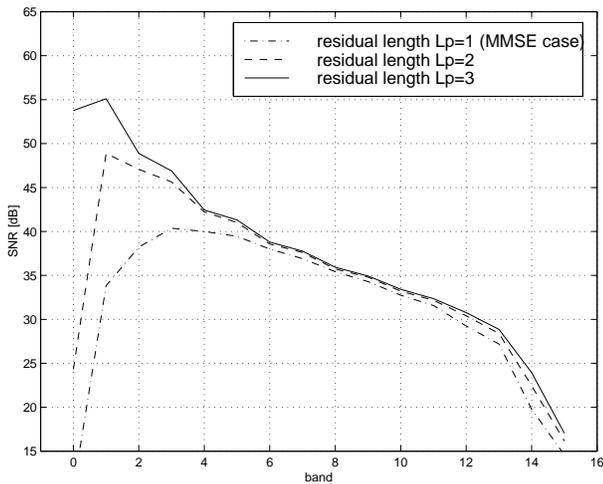


Fig. 4. Signal-to-noise ratios at detector input using a 16-band cosine-modulated filter bank as transmit filters. All bands are loaded.

truncation still results in a noticeable improvement over MMSE equalization for a number of bands. The performance difference between $L_h = 2$ and $L_h = 3$, however, is only marginal in Fig. 5.

From the above example we see that a receiver based on memory truncation receive filters and low-cost Viterbi detectors can yield a significant improvement over MMSE equalization and threshold detection. In general, the amount of SNR improvement of memory truncation over MMSE equalization depends on the channel in question. Significant improvements can be expected whenever it is difficult to equalize a channel because of extreme frequency selectivity.

5 Conclusions

In this paper, optimal receive filter banks for transmultiplexers have been presented. The receive filters are designed in such a way that the overall subchannel impulse responses become truncated to predefined lengths. Using an example of high-speed transmission over copper wires it could be shown that the SNR can be significantly improved over MMSE equalizer banks. In general, the amount of improvement clearly depends on the channel in question, and there may be cases where MMSE approaches work equally well. The design methods presented are applicable to all transmul-

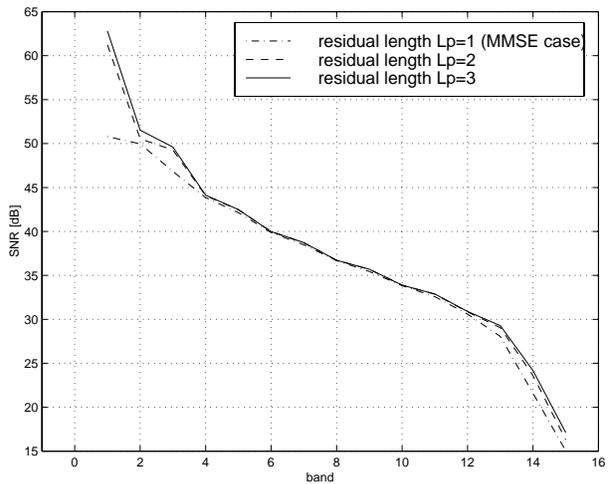


Fig. 5. Signal-to-noise ratios at detector input using a 16-band cosine-modulated filter bank as transmit filters. Only bands 1–15 are loaded.

tiplexing systems where the transmit signal is formed as a linear combination of transmit filter impulse responses with the data symbols being the weights (e.g. DMT, OFDM, CDMA). Extensions of the proposed methods to a joint transmitter/receiver design are under investigation.

References

- [1] B. Hirosaki, "An analysis of automatic equalizers for orthogonally multiplexed QAM systems," *IEEE Trans. Commun.*, vol. 28, pp. 73–83, Jan. 1980.
- [2] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiple access," *IEEE Trans. Commun.*, vol. 33, pp. 665–675, July 1985.
- [3] I. Kalet, "The multitone channel," *IEEE Trans. Commun.*, vol. 37, pp. 119–124, Feb. 1989.
- [4] T. de Couasnon, R. Monnier, and J. B. Rault, "OFDM for digital TV broadcasting," *Signal Processing*, vol. 39, pp. 1–32, Sept. 1994.
- [5] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers, Part I: Unification and optimal designs," *IEEE Trans. Signal Processing*, vol. 47, pp. 1988–2006, July 1999.

- [6] A. Ruiz, J. Cioffi, and S. Kasturia, "Discrete multiple tone modulation with coset coding for the spectrally shaped channel," *IEEE Trans. Commun.*, vol. 40, pp. 1012–1029, June 1992.
- [7] W. Y. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadcasting*, vol. 41, pp. 1–8, Mar. 1995.
- [8] M. G. Bellanger and J. L. Daguët, "TDM–FDM transmultiplexer: Digital polyphase and FFT," *IEEE Trans. Commun.*, vol. 22, pp. 1199–1205, Sept. 1974.
- [9] M. Vetterli, "Perfect transmultiplexers," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Tokyo, pp. 48.9.1–48.9.4, Apr. 1986.
- [10] M. K. Tsatsanis and G. B. Giannakis, "Optimal linear receivers for DS-CDMA systems: A signal processing approach," *IEEE Trans. Signal Processing*, vol. 44, pp. 3044–3055, Dec. 1996.
- [11] A. N. Akansu, P. Duhamel, X. Lin, and M. de Courville, "Orthogonal transmultiplexers in communication: A review," *IEEE Trans. Signal Processing*, vol. 46, pp. 979–995, Apr. 1998.
- [12] R. P. Ramachandran and P. Kabal, "Transmultiplexers: Perfect reconstruction and compensation of channel distortion," *Signal Processing*, vol. 21, pp. 261–274, Nov. 1990.
- [13] L. Vandendorpe, "MMSE equalizers for multitone systems without guard time," in *Proc. 8th Euro. Signal Process. Conf.*, Sept. 1996.
- [14] L. Vandendorpe, L. Cuvelier, F. Deryck, J. Louveaux, and O. van de Wiel, "Fractionally spaced linear and decision-feedback detectors for transmultiplexers," *IEEE Trans. Signal Processing*, vol. 46, pp. 996–1011, Apr. 1998.
- [15] T. Wiegand and N. J. Fliege, "Equalizers for transmultiplexers in orthogonal multiple carrier data transmission," *Annals of Telecommunications*, vol. 52, pp. 39–45, Jan. 1997.
- [16] J. Yang and S. Roy, "On joint transmitter and receiver optimization for multiple-input–multiple-output (MIMO) transmission systems," *IEEE Trans. Signal Processing*, vol. 42, pp. 3221–3231, Dec. 1994.
- [17] N. Al-Dhahir and J. M. Cioffi, "Block transmission over dispersive channels: Transmit filter optimization and realization, and MMSE-DFE receiver performance," *IEEE Trans. Inform. Theory*, vol. 42, pp. 137–160, Jan. 1996.
- [18] T. Li and Z. Ding, "Joint transmitter-receiver optimization for partial response channels based on nonmaximally decimated filterbank precoding technique," *IEEE Trans. Signal Processing*, vol. 47, pp. 2407–2414, Sept. 1999.
- [19] A. Mertins, "Memory truncation and crosstalk cancellation in transmultiplexers," *IEEE Commun. Letters*, vol. 3, pp. 180–182, June 1999.
- [20] D. D. Falconer and F. R. Magee, "Adaptive channel memory truncation for maximum likelihood sequence estimation," *The Bell Systems Technical Journal*, vol. 52, no. 9, pp. 1541–1562, 1973.
- [21] K. D. Kammeyer, "Time truncation of channel impulse responses by linear filtering: A method to reduce the complexity of Viterbi equalization," *Archiv für Elektrotechnik und Übertragungstechnik*, vol. 48, no. 5, pp. 237–243, 1994.
- [22] N. Al-Dhahir and J. M. Cioffi, "Optimum finite-length equalization for multicarrier transceivers," *IEEE Trans. Commun.*, vol. 44, pp. 56–64, Jan. 1996.
- [23] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse response shortening for discrete multi-tone transceivers," *IEEE Trans. Commun.*, vol. 44, pp. 1662–1672, Dec. 1996.
- [24] K. Maxwell, "Asymmetric digital subscriber line: interim technology for the next forty years," *IEEE Commun. Mag.*, vol. 34, pp. 100–106, Oct. 1996.
- [25] S. Trautmann, T. Karp, and N. J. Fliege, "Using modulated filter banks for ISI/ICI-corrupted multicarrier transmission," in *Proc. SPIE Int. Symp. Opt. Science, Eng., and Instrumentation. Session: Wavelet Applications in Signal and Image Processing*, Denver, USA, July 1999.
- [26] H. S. Malvar, "Extended lapped transforms: Fast algorithms and applications," *IEEE Trans. Signal Processing*, vol. 40, Nov. 1992.